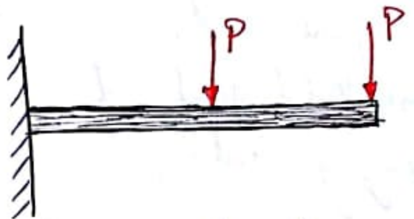
Shear forces & Bending Moments

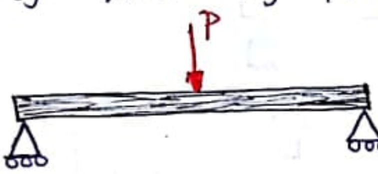
Beam:- A beam is a structural member subjected to system of external forces at right angle to each other.

Types of beams

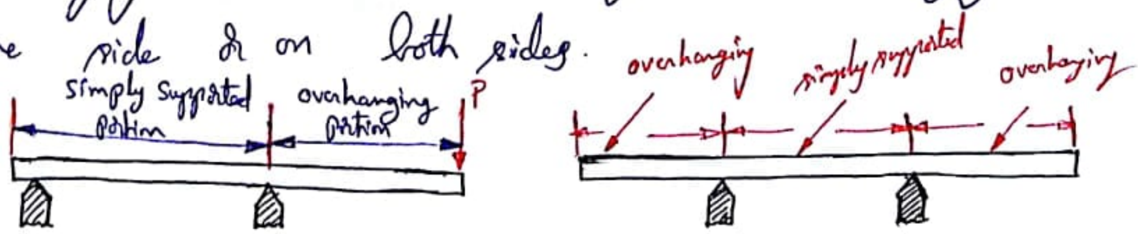
1. Cantilever beam:- If the beam is fixed at one end and free at the other end is known as cantilever beam



2. Simply Supported beam:- A beam is supported at its both ends is known as simply supported beam.



3. Overhanging beam:- A beam having its end portion extends beyond the support is known as overhanging beam. A beam may be overhanging on one side or on both sides.



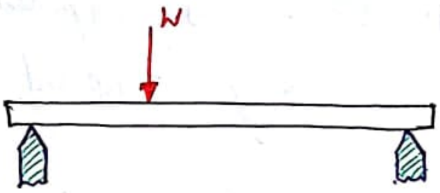
4. Fixed beam:- If the beam is fixed at its both ends, it is called fixed beam

5. <u>Continuous beam</u>:- A beam supported on more than two supports is known as continuous beam.

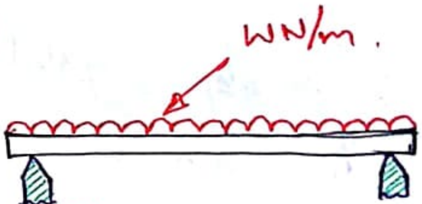
<u>Types of loads</u>.

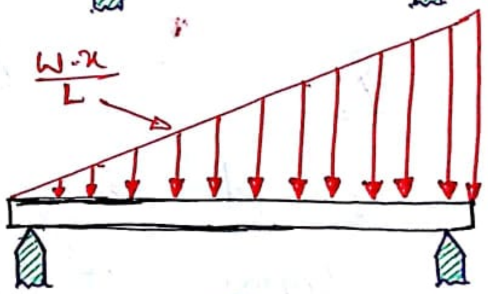
1) Concentrated & point load,
2) uniformly distributed load, and
3) uniformly varying load.

①



②



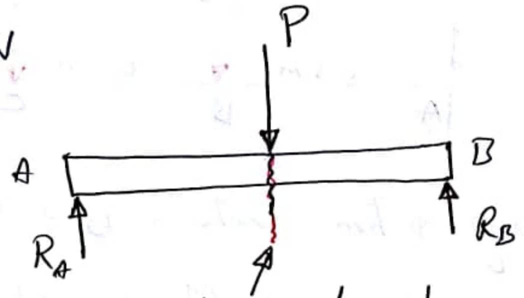


③

Shear force

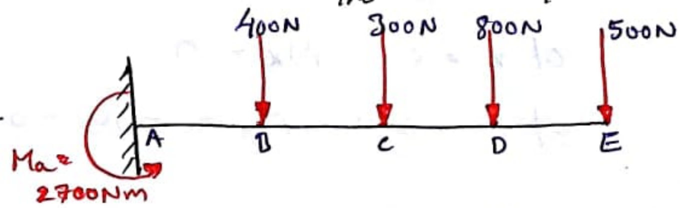
The shear force at the cross section of a beam may be defined as the algebraic sum of all the forces on either side of the section acting normal to the axis of beam.

units → N



The member shear along the load.

Ex:-



→ At any section D & E the shear force is

$S.F.\ S_{DE} = +500N$

→ At any section C&D the shear force is

$S.F.\ S_{CD} = 500 + 800 = +1300N$

→ At any section B&C the shear force is

$S.F\ S_{BC} = 500 + 800 + 300 = +1600N$

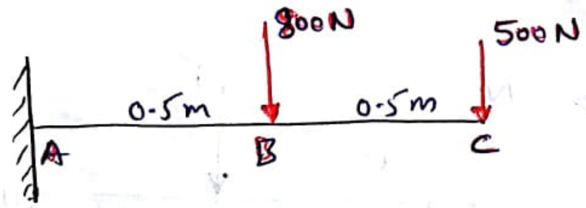
→ At any section A&B the shear force is

$S.F\ S_{AC} = 500 + 800 + 300 + 400 = +2000N$

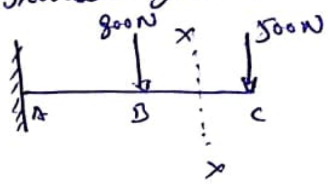
Bending Moment :- The bending moment of the cross-section of a beam may be defined as the algebraic sum of all the moments of the forces on either side of the section.

units → 'N-mm' or 'kN-mm'

Ex:-



At any section between B and C, distance 'x' from C

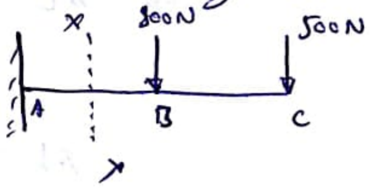


$B.M = M_x = -500x$

at $x = 0$ $M_x = 0$

at $x = 0.5m$ $M_x = -500 \times 0.5 = -250$ Nm

At any section between A and B, distance x from C



$B.M. = M_x = -500x - 800(x - 0.5)$

$= -1300x + 400$

at $x = 0.5m$, $M_x = -1300 \times 0.5 + 400$

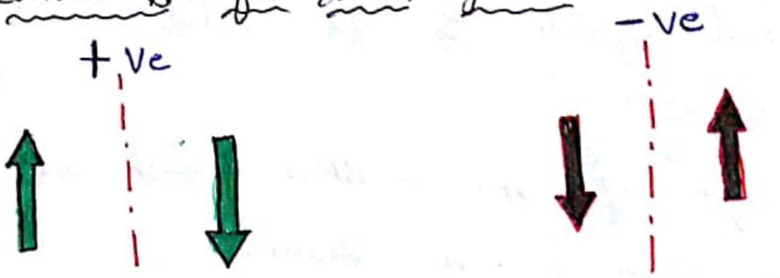
$= -650 + 400$

$= -250$ Nm

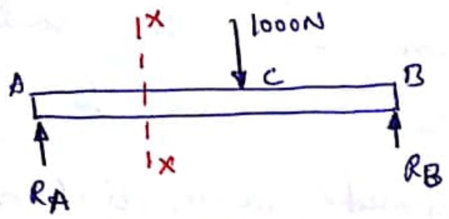
at $x = 1m$, $M_x = -1300 \times 1 + 400$

$= -900$ Nm.

★ Sign Conventions for shear force



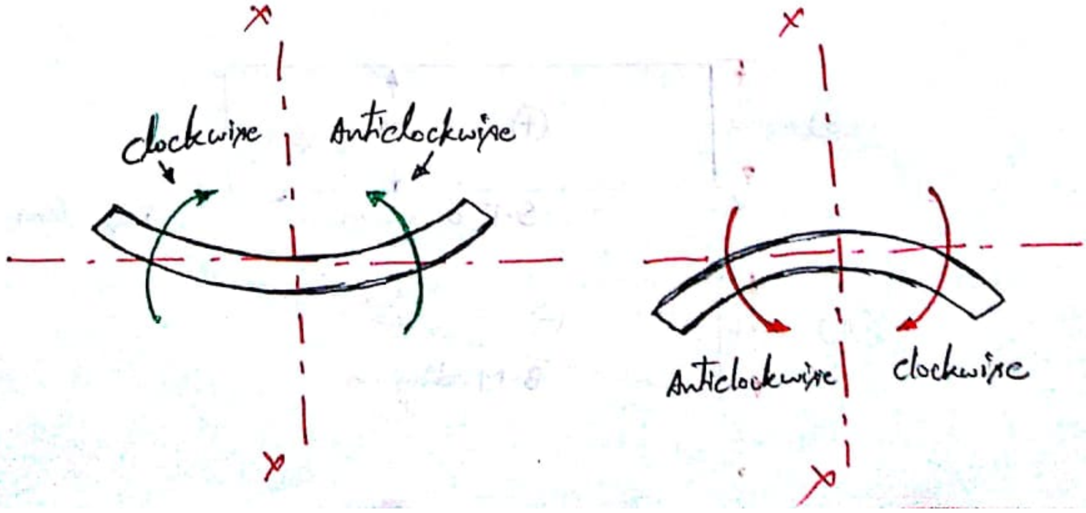
ex:-



$R_A = R_B = 500N$

★ Sign Conventions for bending moment

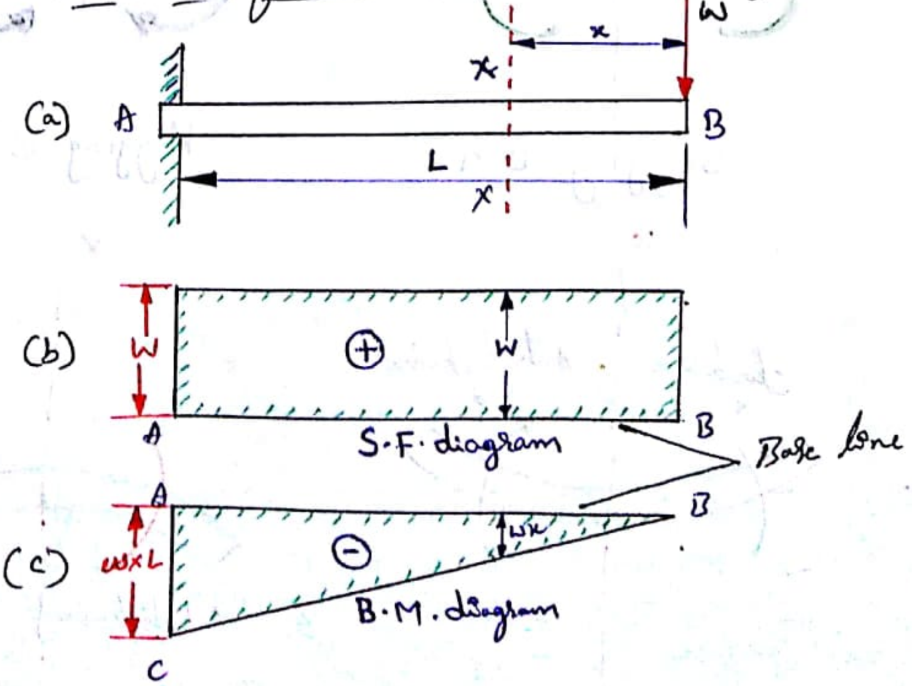
Sagging B.M Hogging B.M



Important points to be noted while drawing SFD & BMD:-

- Length of SFD & BMD must be equal to the span of the beam.
- SFD is drawn below the loaded beam and BMD is drawn below the SFD.
- For simply supported beam, B.M is zero at the supports.
- For cantilever beam, B.M will be zero at free end.
- Calculate SF & BM at all critical points
- If no load is present between two points, then S.F will be constant.

Shear force and Bending moment diagrams for a cantilever with a point load at the free end.



F_x = shear force at x, and
M_x = Bending moment at X

Take a section X at a distance x from the free end. Consider the right portion of the section.

The shear force at X-X is equal to the resultant force acting on the right portion at the given section. But the resultant force acting on the right portion at the section X is W and acting in the downward direction. hence shear force at X is positive

$$F_x = +W$$

SF is constant because no other load is acting between A & B.

<u>Bending Moment Diagram</u>

The cantilever takes the shape of convexity at the top (hogging beam) So, the BM will be negative

The BM at the section X is given by

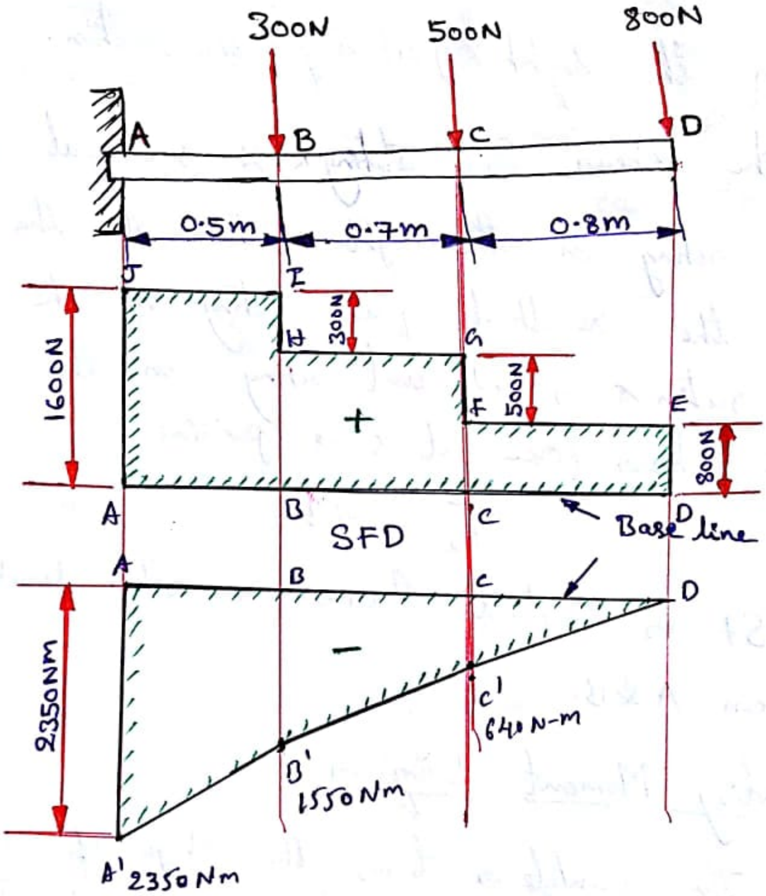
$$M_x = -W \times x$$

BM at any section is proportional to the distance of the section from the free end.

At $x = 0$ ie,, at B, $B \cdot M = 0$
At $x = L$ ie, at A, $B \cdot M = W \times L$

BM follows the straight line law.

Prob:- A cantilever beam of length 2m carries the point loads as shown in the figure. Draw the shear force and B.M diagrams for the cantilever beam.



Shear force Diagram

The shear force at D is +800N. This shear force remains constant between D and C. Similarly for C, B & A

S.F at D, $F_D = +800N$

S.F at C, $F_C = +800 + 500 = +1300N$

S.F at B, $F_B = +800 + 500 + 300 = +1600N$

S.F at A, $F_A = +1600N$

Bending Moment Diagram

The bending moment at D is zero:

i) the bending moment at any section between C&D at a distance x and D is given by.

$M_x = -800x$ which follows a straight line law.

At C, the value of $x = 0.8\,m$

$$M_C = -800 \times 0.8$$
$$= -640\ Nm$$

ii) B.M at any section b/w B & C

$$M_C = -800x - 500(x - 0.8)$$

At C, $x = 0.8$

$$= -800 \times 0.8 - 500(0.8 - 0.8)$$
$$= -640\ Nm$$

At B, $x = 1.5m$

$$M_B = -800 \times 1.5 - 500(1.5 - 0.8)$$
$$= -1200 - 350$$
$$= -1550\ Nm$$

iii) B.M at any section b/w B & A

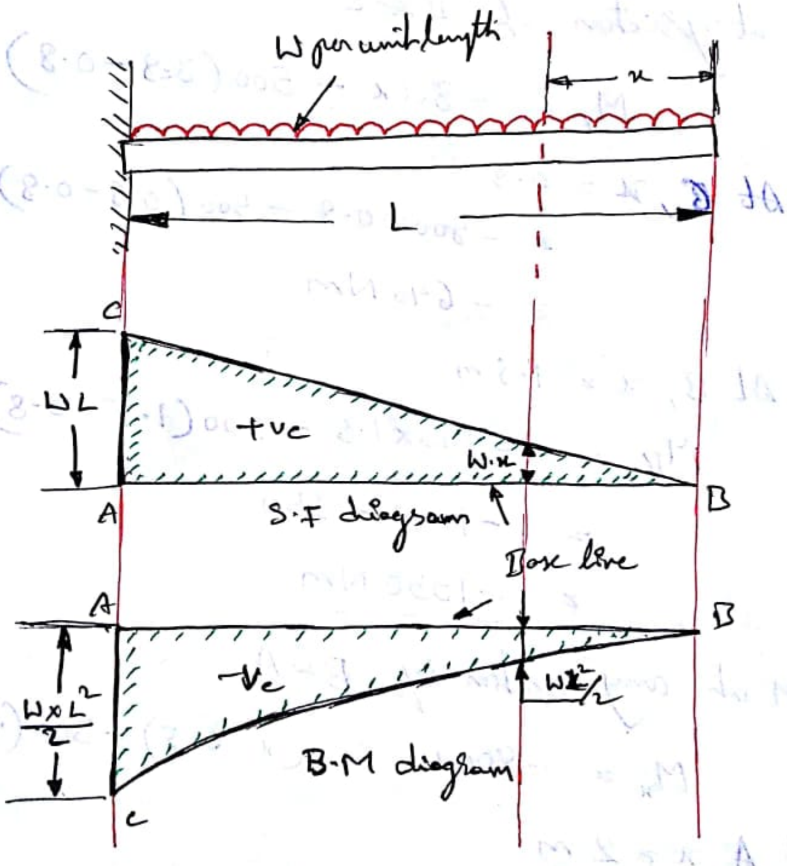
$$M_x = -800x - 500(x - 0.8) - 300(x - 1.5)$$

At A, $x = 2\,m$

$$M_A = -1600 - 600 - 150$$
$$= -2350\ Nm$$

Shear force and bending moment diagrams for a Cantilever with a uniformly distributed load

Figure shows a cantilever of length L fixed at A and carrying a uniformly distributed load of w per unit length over the entire length of the cantilever.



SFD

Take a section X at a distance of x from the free end B

Let F_x = Shear force at X and

M_x = Bending moment at X

Consider shear force at the section x will be equal to the resultant force acting on the right portion of the section. Resultant force on the right portion acting downwards is considered positive. Hence shear force at x is positive

$$\boxed{F_x = +w x}$$

The above equation shows that the shear force follows a straight line law.

at B, $x=0$ and hence $\boxed{F_x = 0}$

at A, $x=L$ and hence $\boxed{F_x = w \cdot L}$

BMD

Uniformly distributed load over a section is converted into point load acting at the C.G of the section.

The bending moment at the section x is given by

M_x = − Total load on right portion × Distance of C.G. of right portion from x.

$$= -(w \cdot x) \cdot \frac{x}{2} = -w\frac{x^2}{2}; \quad \boxed{M_x = w \cdot \frac{x^2}{2}}$$

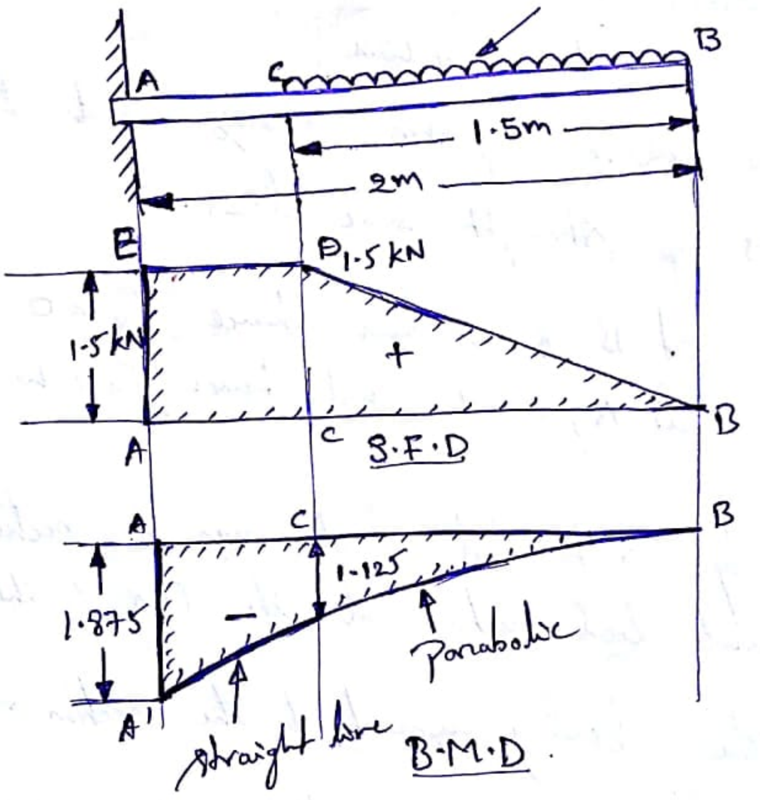
It is clear that B.M. at any section is proportional to the square of the distance of the section from the free end. This follows a parabolic law.

at B, $x=0$ hence $\boxed{M_x = 0}$

at A, $x=L$ hence $\boxed{M_x = -w \cdot \frac{L^2}{2}}$

Prob: A cantilever of length 2.0 m carries a uniformly distributed load of 1 kN/m run over a length of 1.5 m from the free end. Draw the shear force and bending moment diagrams for the cantilever.

Sol: UDL Problem



Shear force Diagram

Consider any section between C and B a distance of x from the free end B. The shear force at the section is given by

$$F_x = w \cdot x$$

(+ve sign is due to downward force on right portion of the section)

At B, $x = 0$ hence

$$F_x = 0$$

At C, $x = 1.5$ hence

$$F_x = 1.0 \times 1.5 = 1.5\,kN$$

The shear force follows a straight line law between C and B. As there is no load b/w A & C shear force is represented by horizontal line.

$$F_B = 0$$

$$F_C = 1.5\,kN \text{ and}$$

$$F_A = 1.5\,kN$$

Bending Moment Diagram

i) The bending moment at any section between C and B at a distance x from the free end B is given by

$$M_x = -(Wx)\cdot\frac{x}{2}$$

$$= -\left(1\cdot\frac{x^2}{2}\right) = \frac{x^2}{2} \quad \text{(i)}$$

(the bending moment will be negative as for the right portion of the section the moment of load at x is clockwise).

at B, $x=0$ hence $M_B = -\frac{0^2}{2} = 0$

at C, $x=1.5$ hence $M_C = -\frac{1.5^2}{2} = -1.125\,Nm$

Bending moment varies according to parabolic law between C and B.

ii) The bending moment at any section between A and C at a distance x from the free end B is obtained as: (where x varies from 1.5m to 2.0m)

Total load due to UDL $= \omega \times 1.5 = 1.5$ kN.

This load is acting at a distance of $\frac{1.5}{2} = 0.75$m from the free end B & at a distance of $(x-0.75)$ from any section between A and C.

$\therefore$ Moment of this load at any section between A and C at a distance x from free end.

$= (\text{load due to UDL}) \times (x-0.75)$

$$M_x = -1.5 \times (x-0.75) \quad \text{—— (ii)}$$

(-ve sign is due to clockwise moment for right portion)

B.M follows a straight line law between A and C.

(i) At C, $x = 1.5$m hence

$$M_C = -1.5(1.5-0.75)$$
$$= -1.125 \text{ Nm}$$

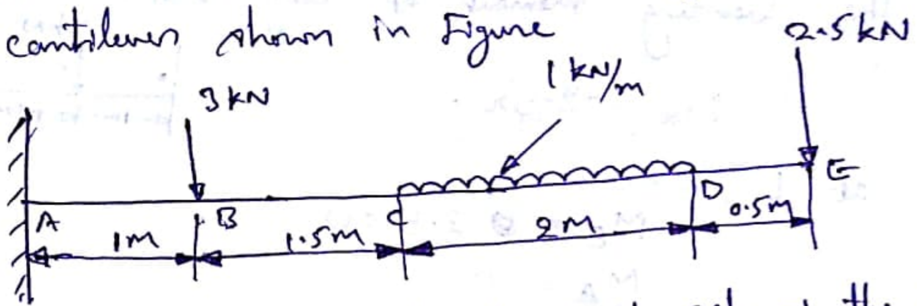
At A, $x = 2$m hence

$$M_A = -1.5(2-0.75)$$
$$= -1.875 \text{ Nm}$$

B to C' — parabolic curve

A' to C' — straight line.

Prob: Draw the shear force and bending moment diagrams for the cantilever shown in Figure



S.F.D (load is acting at the right side of the section so SF is +ve)

Shear force from D to E.
As it is point load the S.F follows a straight line horizontal.

$$F_E = 2.5 \text{ kN}$$

$$F_D = 2.5 \text{ kN}$$

Shear force from C to D.
As it is UDL the S.F follows a straight line inclined (increasing towards fixed end)

$$F_x = 2.5 + 1 \times x$$

at D; $x = 0$, $F_D = 2.5 + 0 = 2.5$ kN

at C; $x = 2$, $F_C = 2.5 + 1 \times 2 = 4.5$ kN

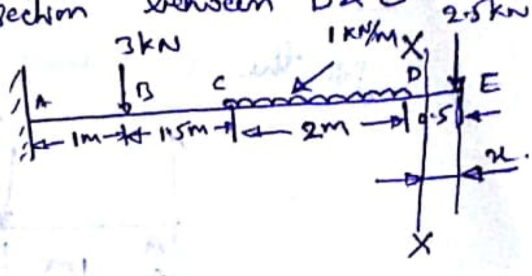
Shear force from B to C.
There is no load acting b/w B & C, So the load at C Continuous to B.

$$F_B = 4.5 \text{ kN}$$

Shear force from A to B
As it is point load the S.F follows a straight line horizontal. $F_A = 4.5 + 3 = 7.5$ kN.

B.M.D

The bending moment at any section between D & E



$M_x = -2.5x$.

at E, $x = 0\,m$

$M_E = 2.5 \times 0$

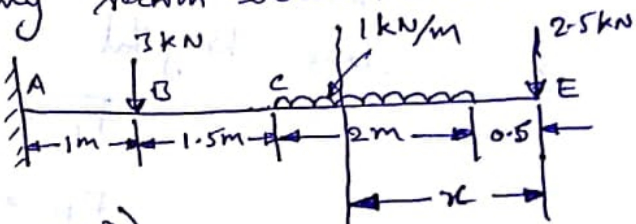
$M_E = 0$

at D, $x = 0.5\,m$

$M_D = -2.5 \times 0.5$

$M_D = -1.25\ kN{\cdot}m$

the bending moment at any section between C & D



$M_x = -(2.5x + 1 \times 2(x - 1.5))$

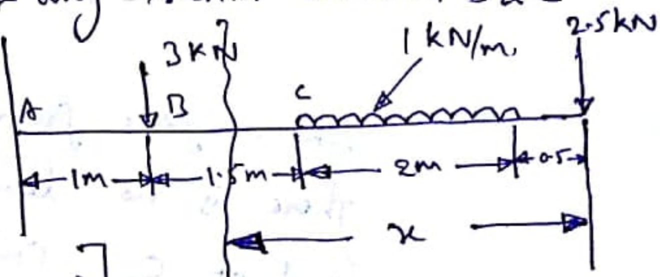
at C, $x = 2.5\,m$.

$= -(2.5 \times 2.5 + 2(1))$

$= -(6.25 + 2)$

$M_C = -8.25\ kNm$.

The bending moment at any section between B & C



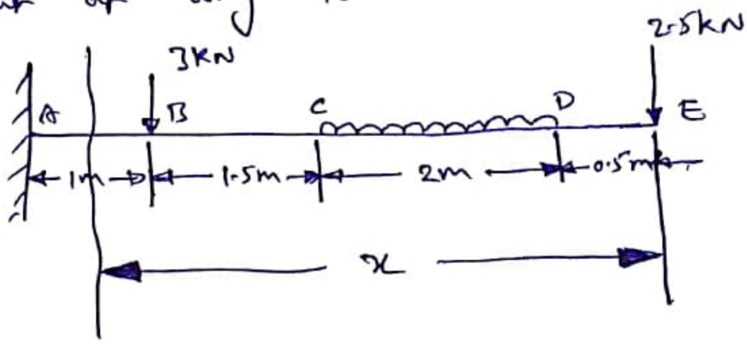
$M_x = -\left[2.5 \times x + 1 \times 2(x - 1.5)\right]$

at B, $x = 4\,m$.

$M_B = -\left[2.5 \times 4 + 2(4 - 1.5)\right]$

$= -15\ kN{-}m$

The bending moment at any section between A & B



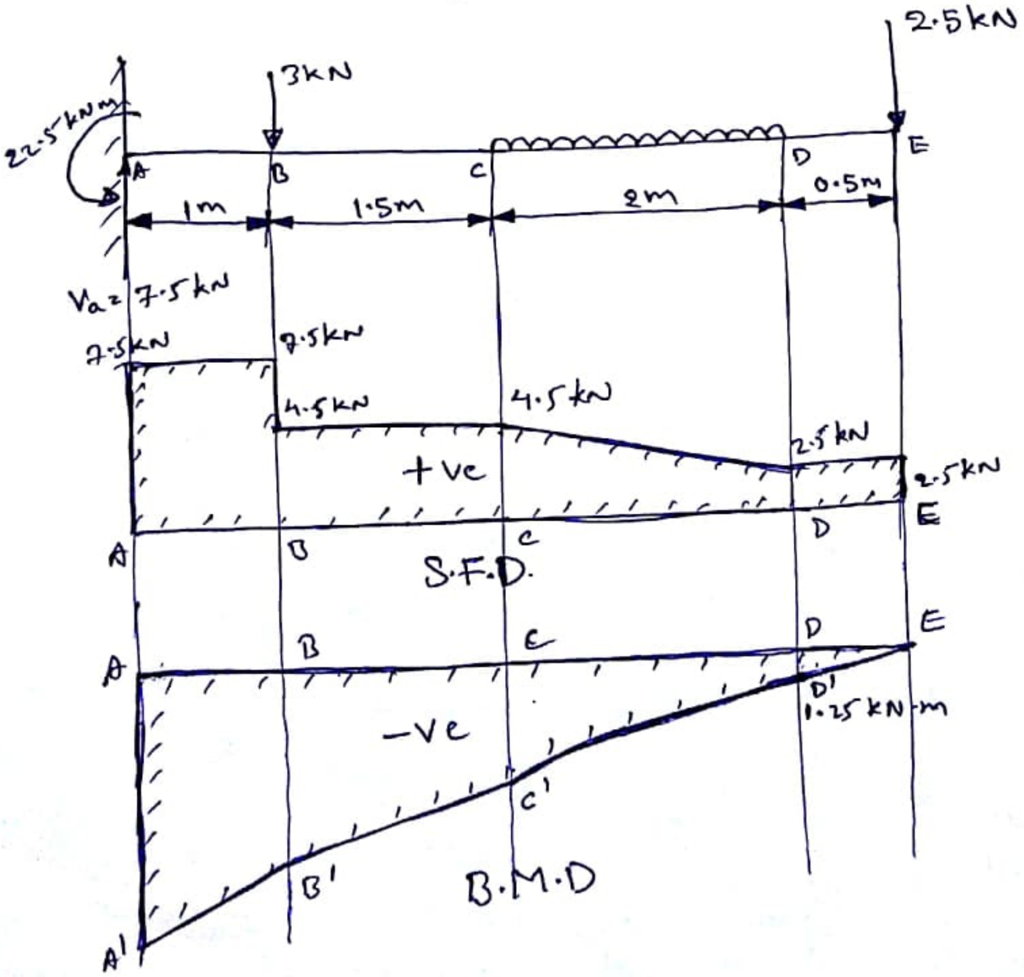
$$M_x = -\left[2.5x + 1\times2\,(x-1.5) + 3\,(x-4)\right]$$

at A, $x = 5m$.

$$M_A = -\left[2.5\times5 + 2(5-1.5) + 3(5-4)\right]$$

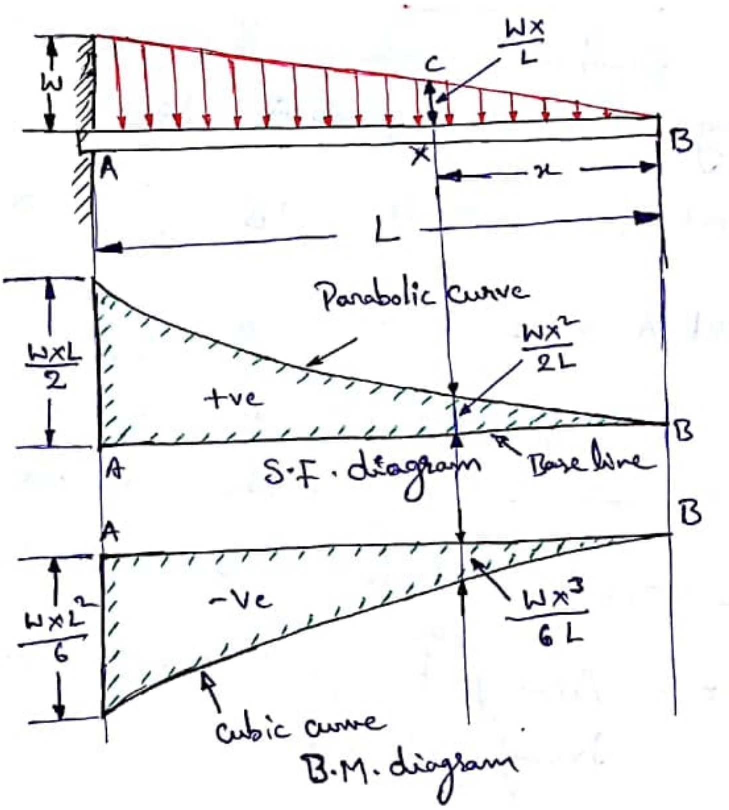
$$= -\left[12.5 + 7 + 3\right]$$

$$= -22.5 \text{ kN-m.}$$



Shear force and bending moment diagrams for a Cantilever carrying a gradually varying load.

A cantilever of length L fixed at A and carrying a gradually varying load from zero at the free end to w per unit length at the fixed end.



Take a section x at a distance x from the free end B.

Let F_x = Shear force at the section x, and

M_x = Bending moment at the section x

The rate of loading is zero at B and is w per metre run at A.

Hence the rate of loading for a length of x will be $\frac{w}{L} \times x$ per unit length.

The shear force at a section X at a distance x from free end, is given by.

F_x = Area of triangle BCX

$$= \frac{XB \cdot XC}{2} = \frac{x\left(\frac{wx}{L}\right)}{2}$$

$$\boxed{F_x = \frac{wx^2}{2L}} \quad \text{——— (i)}$$

The equation (i) shows that the S.F varies according to the parabolic law.

At B, $x = 0$ hence $F_B = \frac{w \times 0^2}{2L} = 0$

At A, $x = L$ hence $F_A = \frac{w \cdot L^2}{2L} = \frac{wL}{2}$

The bending moment diagram at the section X at a distance x from the free end B is given by

M_x = −(Total load for a length x) × Distance of the load from X

= − Area of triangle BCX × Distance of CG of the triangle from X

$$= -\left(\frac{wx^2}{2L}\right) \times \frac{x}{3} = -\frac{wx^3}{6L}$$

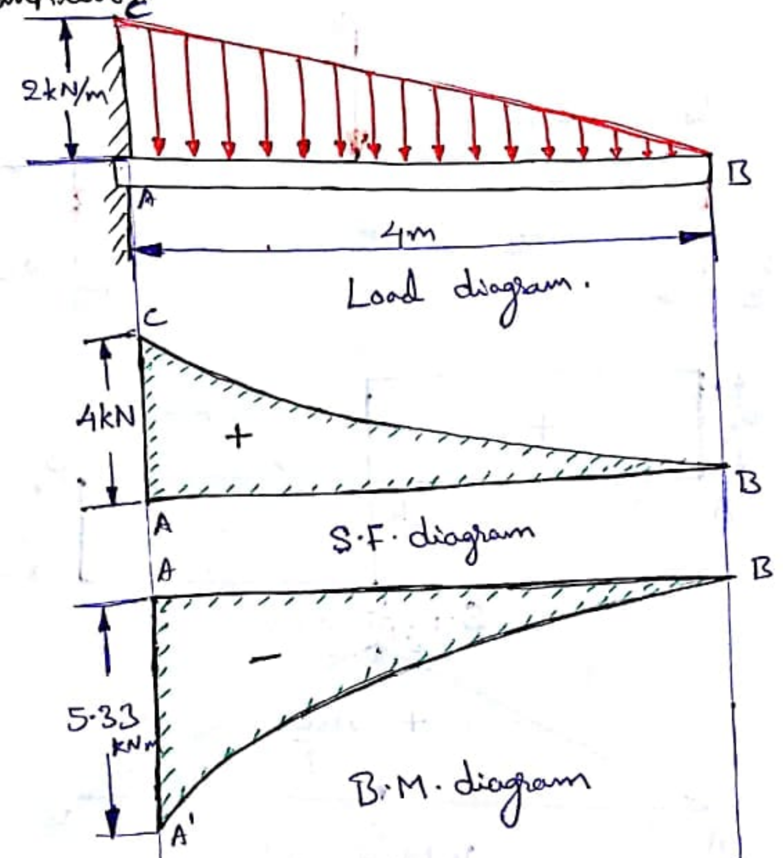
$$\boxed{M_x = -\frac{wx^3}{6L}} \quad \text{——— (ii)}$$

The equation (ii) shows that the BM. varies according to the cubic law.

At B, $x = 0$ hence $M_B = \frac{-w \times 0}{6L} = 0$

At A, $x = L$ hence $M_A = \frac{-w \cdot L^3}{6L} = -\frac{wL^2}{6}$

Prob:- A cantilever of length 4 m carries a gradually varying load, zero at the free end to 2 kN/m at the fixed end. Draw the SF and B.M diagrams for the cantilever.



SFD

The shear force is zero at B.

Shear force at C = area of load diagram ABC

$$SF \text{ at } C = \frac{4\times 2}{2} = 4\text{ kN}$$

The shear force between A and B varies according to Parabolic law.

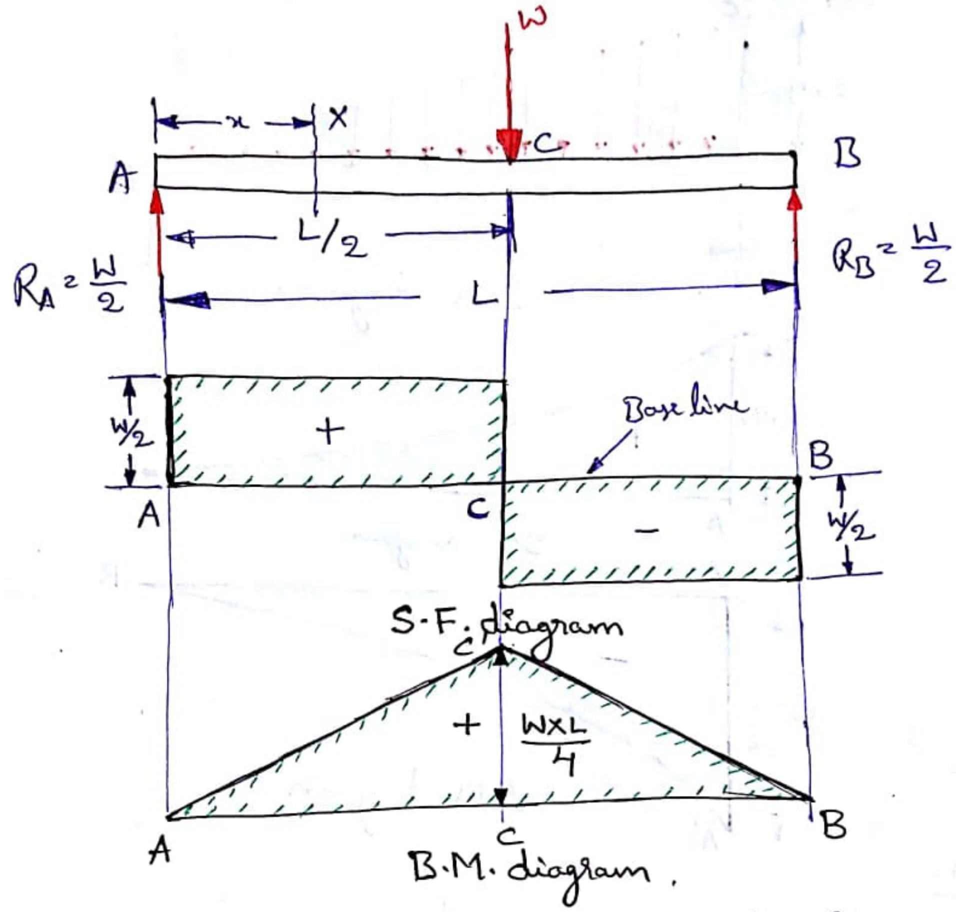
BMD

The bending moment at B is zero. The bending moment at A is equal to $-\frac{wL^2}{6}$

$$M_A = -\frac{wL^2}{6} = -\frac{2\times 4^2}{6} = -5.33\text{ kNm}.$$

The B.M. between A and B varies according to cubic law.

Shear force and bending moment diagrams for a simply supported beam with a point load at mid-point.



The reactions at the support will be equal to W/2 as the load is acting at the middle point of the beam. Hence $R_A = R_B = \frac{W}{2}$

<u>SFD</u>

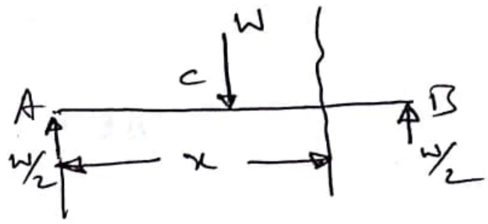
Take a section x at a distance x from the end A b/w A & C

The resultant force $\frac{W}{2}$ on the left portion acting upwards is considered positive.

$$F_x = +\frac{W}{2}$$

Hence the shear force between A and C is constant and equal to $+\frac{W}{2}$

Consider any section between C and B at a distance x from end A



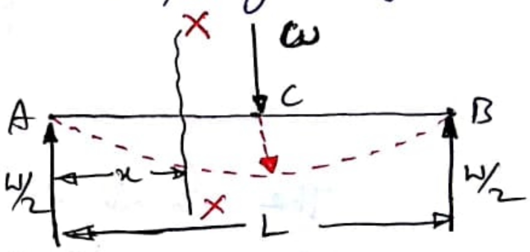
The resultant force on the left portion will be.

$$\left[\frac{W}{2} - W\right] = -\frac{W}{2}$$

The shear force $-\frac{W}{2}$ is constant between C and B

BMD

The bending moment at any section between A and C at a distance of x from the end A, is given by.



$$M_x = R_A x$$
$$= +\frac{W}{2} x.$$

B.M is positive as for the left portion of the section the moment of all forces is clockwise. The beam take concavity at top.

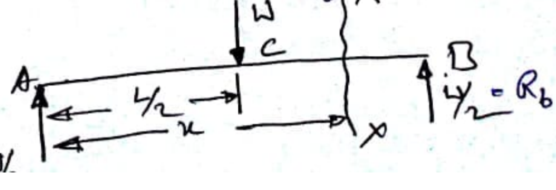
At A, $x = 0$

$$M_A = 0$$

At B, $x = L/2$

$$M_B = \frac{WL}{2 \times 2} = \frac{WL}{4}$$

The bending moment at any section between C and B. at a distance of x from the end A, is given by.



$$M_x = \frac{W}{2} \times x - W\left(x - L/2\right).$$

$$M_x = \frac{W}{2}x - Wx + \frac{WL}{2}$$

$$= \frac{Wx - 2Wx + WL}{2}$$

$$= \frac{WL - Wx}{2}$$

At C, $x = \frac{L}{2}$ hence

$$M_C = \frac{WL}{2} - \frac{WL}{4}$$

$$= \frac{2WL - WL}{4}$$

$$\boxed{M_C = \frac{WL}{4}}$$

At B, $x = L$ hence

$$M_B = \frac{WL}{2} - \frac{WL}{2}$$

$$= 0$$

The bending moment is maximum at the middle point C, where the shear force changes its sign.

Shear force and bending moment diagrams for a simply supported beam with an eccentric point load.

First calculate the reactions, by taking moments about A & about B.

Taking moments of the forces on the beam about A, we get

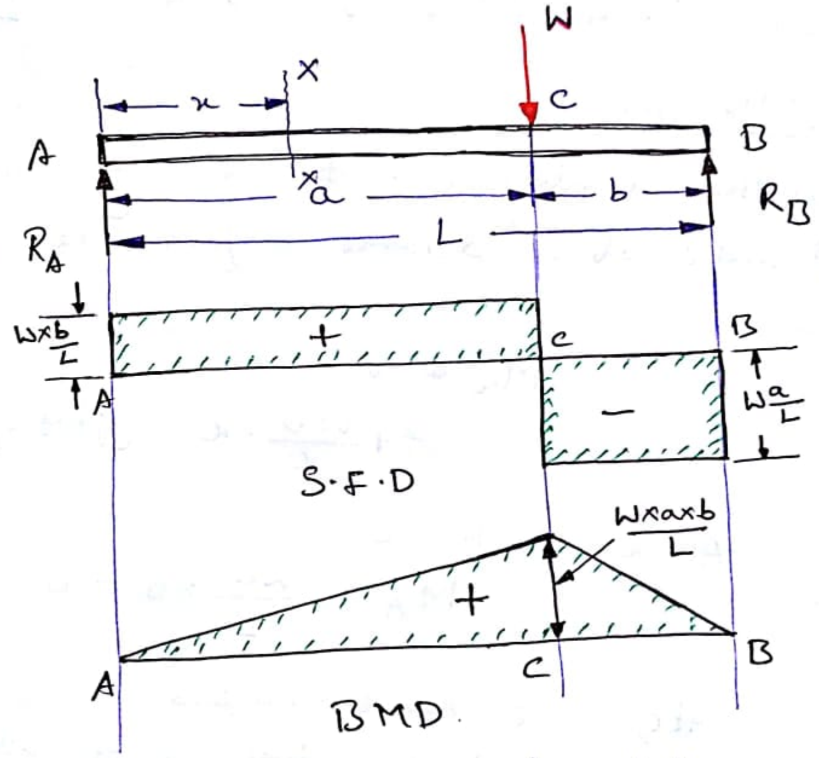
$$R_B \times L = W \times a.$$

$$R_B = W\frac{a}{L}$$

$$R_A + R_B = W$$

$$R_A = W - R_B$$
$$= W - \frac{W \cdot a}{L}$$
$$= W\left(\frac{L-a}{L}\right). \quad (L-a=b).$$
$$R_A = \frac{Wb}{L}$$

8



<u>SFD</u> Consider any section x at a distance x from end A. between A & C

$$F_x = R_A = +\frac{Wb}{L}$$

The SF will be positive as the resultant force on the left portion of the section is acting upwards.

SF between A & C is constant and equal to $\frac{Wb}{L}$

Consider any section X at a distance x from end A between C & B.

Resultant force $R_A - W$

$$= \frac{Wb}{L} - W$$

$$= W\left(\frac{b-L}{L}\right)$$

$$= -W\left(\frac{L-b}{L}\right)$$

$$= -\frac{Wa}{L}$$

SF between C & B is constant $-\frac{Wa}{L}$.

BMD

i) The bending moment at any section between A and C at a distance x from the end A is given by

$$M_x = R_a \times x.$$

$$= +\frac{Wb}{L} \cdot x.$$ (plus sign due to sagging).

At A, $x = 0$ hence

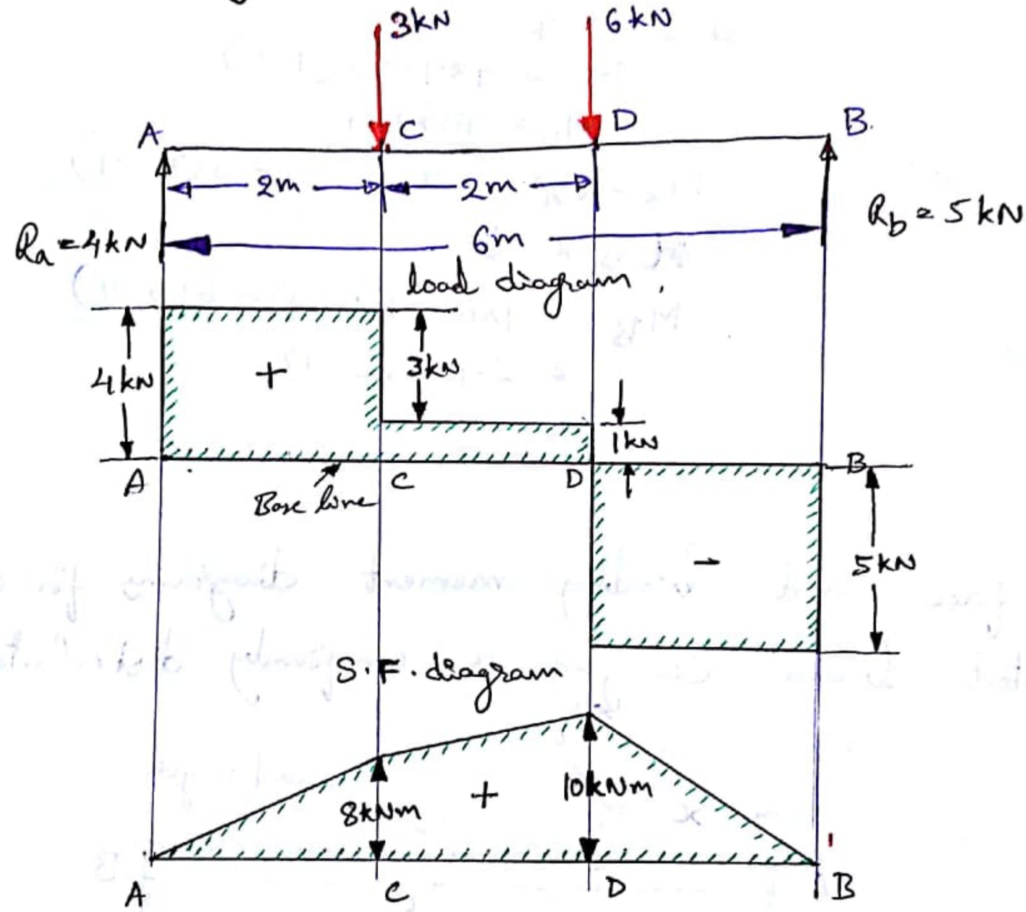
$$M_A = \frac{Wb}{L} \times 0 = 0$$

At C $x = a$ hence

$$M_C = \frac{Wb}{L} \cdot a = \frac{W \cdot a \cdot b}{L}$$

Hence the B.M will increase from zero ~~at~~ A to $\frac{Wab}{L}$ at C by a straight line law. The bending moment is zero at B.

A simply supported beam of length 6m, carries point load of 3kN and 6kN at distances of 2m and 4 m from the left end. Draw the shear force and bending moment diagrams for the beam.



Taking moments of the force about A, we get.

$$R_B \times 6 = 3\times 2 + 6\times 4$$

$$= 30$$

$$R_B = \frac{30}{6} = 5\,kN$$

$$R_A + R_B = 3+6$$

$$R_A = 9-5 = 4\,kN$$

<u>SFD</u>

$$F_A = R_a = 4\,kN$$

$$F_c = 4-3 = 1\,kN$$

$$F_D = 4-3-6 = -5\,kN$$

$F_B = -5\,kN$

BMD

b/w A&C B.M at A, $M_x = R_A \times x$.

at A, $x=0$ $M_A = 0$

at C, $x=2$ $M_C = 4\times 2 = 8\,kN{\cdot}m$

b/w C&D $M_x = R_A \times x - 3(x-2)$.

at D, $x=4$

$M_D = 4\times 4 - 3(4-2)$.

$M_D = 10\,kNm$.

b/w D&B $M_x = R_A \times x - 3(x-2) - 6(x-4)$

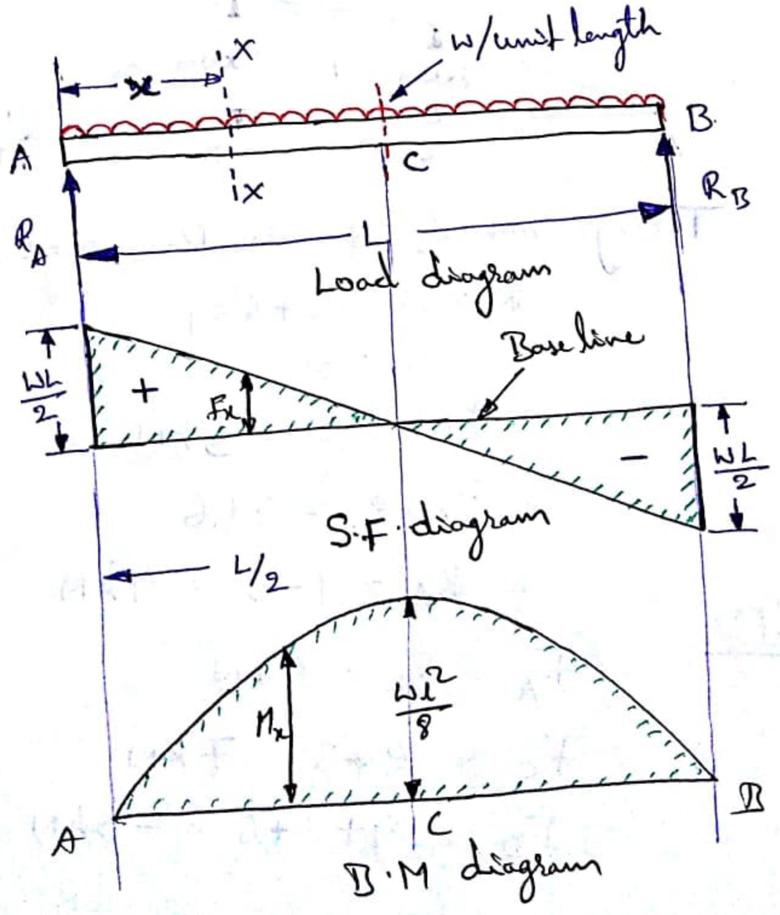
at B, $x=6$

$M_B = 4\times 6 - 3(6-2) - 6(6-4)$

$= 24 - 12 - 12$

$= 0$

Shear force and bending moment diagrams for a simply supported beam carrying a uniformly distributed load.



$$R_A = R_B = \frac{wL}{2}$$

SFD

Consider any section x at a distance x from the left end A.

$$F_x = +R_A - w \cdot x$$

$$= \frac{wL}{2} - w \cdot x \quad \text{—————} \quad (i)$$

From equation (i), it is clear that the shear force varies according to straight line law.

At A, $x = 0$ hence $F_A = +\frac{wL}{2} - \frac{w0}{2} = +\frac{wL}{2}$

At B, $x = L$ hence $F_B = +\frac{wL}{2} - wL = -\frac{wL}{2}$

At C, $x = L/2$ hence $F_C = +\frac{wL}{2} - \frac{wL}{2} = 0$

BMD

The bending moment at the section x at a distance x from left end A is given by,

$$M_x = +R_A x - w \cdot x \frac{x}{2}$$

$$= \frac{wL}{2} \cdot x - w\frac{x^2}{2} \quad \text{—————} \quad (ii)$$

From the equation (ii), it is clear that B.M varies according to parabolic law.

The value of B.M at different points:

At A, $x = 0$ hence

$$= \frac{wL}{2} 0 - w\frac{0}{2} = 0$$

At, B, $x = L$ hence.

$$M_B = \frac{wL}{2}L - \frac{wL^2}{2} = 0$$

At C, $x = L/2$ hence

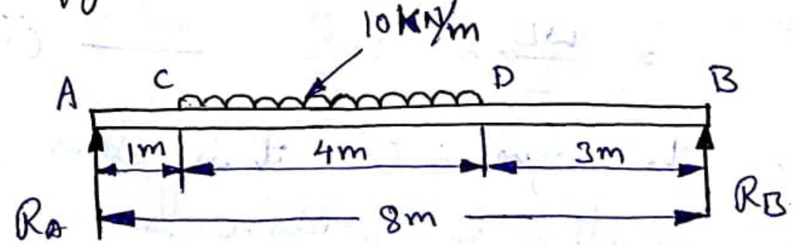
$$M_C = \frac{wL}{2}\frac{L}{2} - \frac{w}{2}\frac{L^2}{4}$$

$$= \frac{wL^2}{4} - \frac{wL^2}{8}$$

$$\boxed{M_C = +\frac{wL^2}{8}}$$

Thus the B.M increases according to parabolic law from zero at A to $+\frac{wL^2}{8}$ at the middle point of the beam and from this value the B.M decreases to zero at B according to the parabolic law.

Prob:- Draw the shear force and B.M diagrams for a simply supported beam of length 8m and carrying a uniformly distributed load of 10 kN/m for a distance of 4m as shown in figure.



Taking moments of the forces about A, we get

$$R_B \times 8 = 10 \times 4 \times \left(1 + \frac{4}{2}\right)$$

$$= 120$$

$$R_B = 15 \text{ kN}$$

$$R_A + R_B = 10 \times 4$$
$$R_A = 40 - 15$$
$$R_A = 25\,kN.$$

S.F.D

The shear force at A is

$$\boxed{F_A = +25\,kN}$$

Shear force at C is

$$F_x = 25 - 10(x-1)$$

at C, $x = 1$

$$\boxed{F_C = 25\,kN}$$

at D, $x = 5$

$$F_D = 25 - 10(5-1)$$
$$= 25 - 40$$
$$\boxed{F_D = -15\,kN}$$

Shear force at B is

$$\boxed{F_B = -15\,kN}$$

B.M.D

B.M between A & C.

$$M_x = R_A x.$$

at A, $x = 0$

$$M_A = 0$$

at C, $x = 1$

$$M_C = 25 \times 1$$
$$= 25\,kNm.$$

B.M between C & D.

$$M_x = R_A x - 10 \frac{4}{2}(x-1)$$

at D, $x = 5$

$$M_D = 25 \times 5 - 10 \times 2 \times 4$$
$$= 125 - 80 = 45\,kNm.$$

B.M b/w D & E

$$M_x = R_a \times x - 10 \times 4 \left(x - \left(\frac{4}{2}+1\right)\right)$$

at E, $x = 8$.

$$M_E = 25 \times 8 - 10 \times 4 (8-3)$$

$$= 200 - 200$$

$$= \underline{\underline{0}}$$

But the critical point E is situated at Maximum bending moment where shear force is zero ($F_x = 0$)

$$F_x = 25 - 10(x-1)$$

$$0 = 25 - 10x + 10$$

$$10x = 35$$

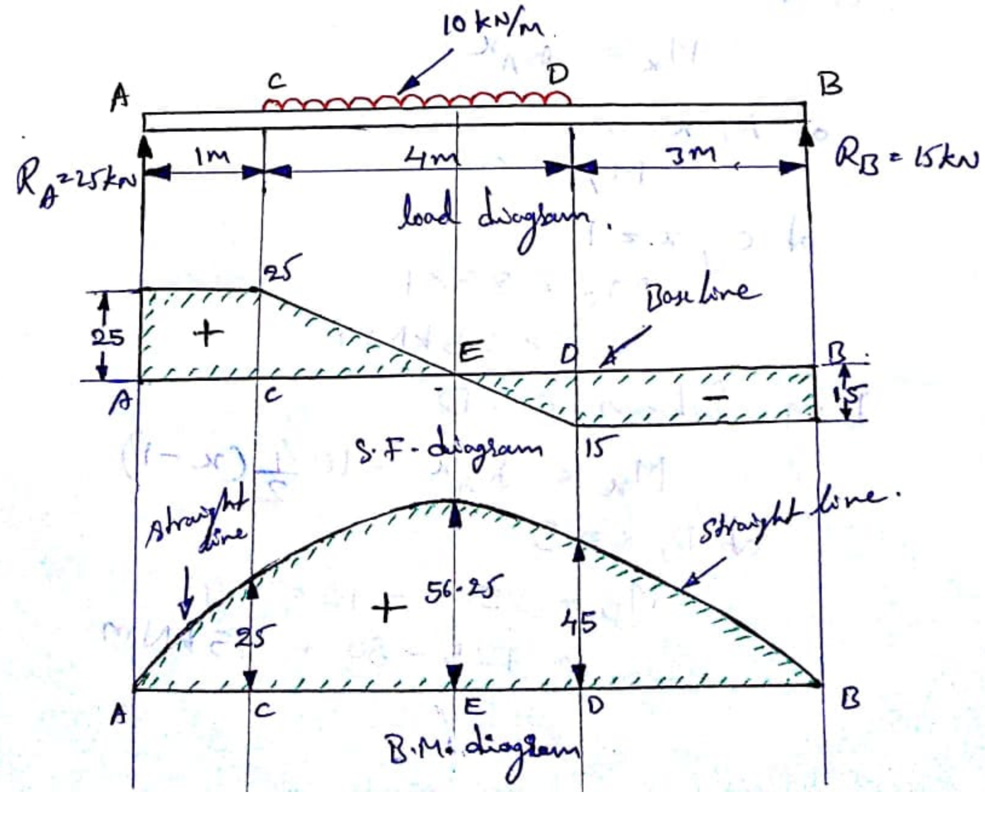
$$x = 3.5 \text{ mm.}$$

B.M at E, is

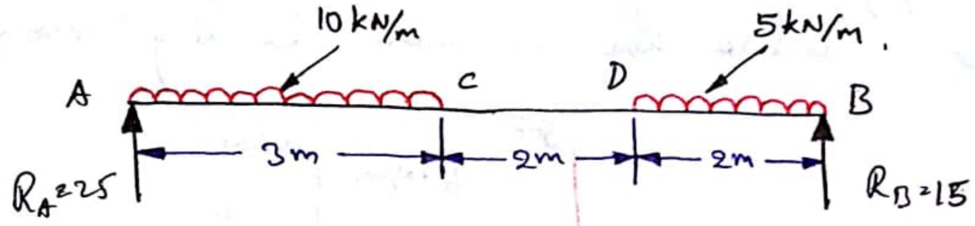
$$M_E = 25 \times 3.5 - 10 \times \frac{(3.5-1)^2}{2}$$

$$= 87.5 - 31.25$$

$$M_E = 56.25 \text{ kNm.}$$



Prob: Draw the S.F and B.M. diagrams of a simply supported beam of length 7m carrying uniformly distributed loads as shown in figure.



SFD

$F_A = 25$ kN

$F_C = 25 - 10 \times 3 = -5$ kN

$F_D = -5$ kN

$F_B = 25 - 30 - 5 \times 2 = -15$ kN

BMD

M_A & $M_B = 0$

$M_C = R_A \times 3 - \overset{5}{\cancel{10}} \times 3 \left(\frac{3}{2}\right)$

$= 25 \times 3 - 45$

$= 30$ kNm.

$M_D = R_A \times 5 - 10 \times 3 \times 3.5$

$= 125 - 105$

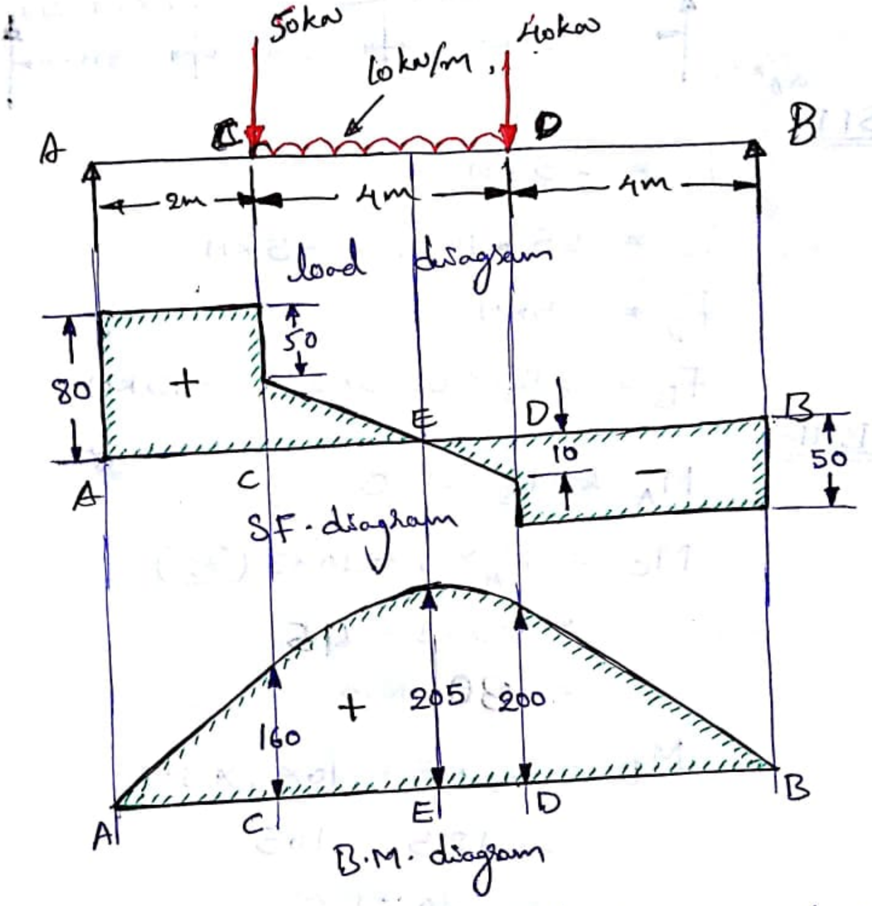
$= 20$ kNm.

$M_B = R_A \times 7 - 10 \times 3 \times 5.5 - 5 \times 2 \times 1$

$= 175 - 165 - 10$

$= \underline{0}$

Prob: A simply supported beam of length 10m, carries the UDL and two point loads as shown in figure. Draw SFD and BMD for the beam, also calculate the maximum bending moment.



Taking moments of all forces about A, we get R_A & R_B.

$$R_B \times 10 = 50\times2 + 10\times4\times\left(2+\frac{4}{2}\right) + 40\times6$$

$$= 100 + 160 + 240$$

$$= 500$$

$$R_B = 500/10 = 50\,kN$$

$$R_A + R_B = 50 + 40 + 10\times4$$

$$R_A = 130 - 50$$

$$R_A = 80\,kN$$

SF. diagram

The SF at A,

$$F_A = R_A = 80\,kN$$

SF remains constant between A and C

SF at C,

$$F_C = 80 - 50 = 30\,kN$$

SF at D

$$F_D = 80 - 50 - 10 \times 4.$$
$$= 80 - 90 = -10\,kN$$

but point load acts at D, so

$$F_D = -10 - 40 = -50\,kN$$

$F_D = -50\,kN$ remains constant b/w D & B.

The shear force at point E is zero, Let the distance of E from point A is x.

Now shear force at $E = R_A - 50 - 10(x-2)$

$$= 80 - 50 - 10x + 20$$

but shear force at $E = 0$

$$50 - 10x = 0$$
$$x = \frac{50}{10} = 5m$$

B.M. diagram

B.M at A is zero

B.M at B is zero

B.M at C,

$$M_C = R_A \times 2$$
$$= 80 \times 2 = 160\,kN\text{-}m$$

B.M at D,

$$M_D = R_A \times 6 - 50 \times 4 - 10 \times 4 \times \frac{4}{2}.$$
$$= 200\,kNm$$

At E, $x = 5m$ and hence B.M at E,

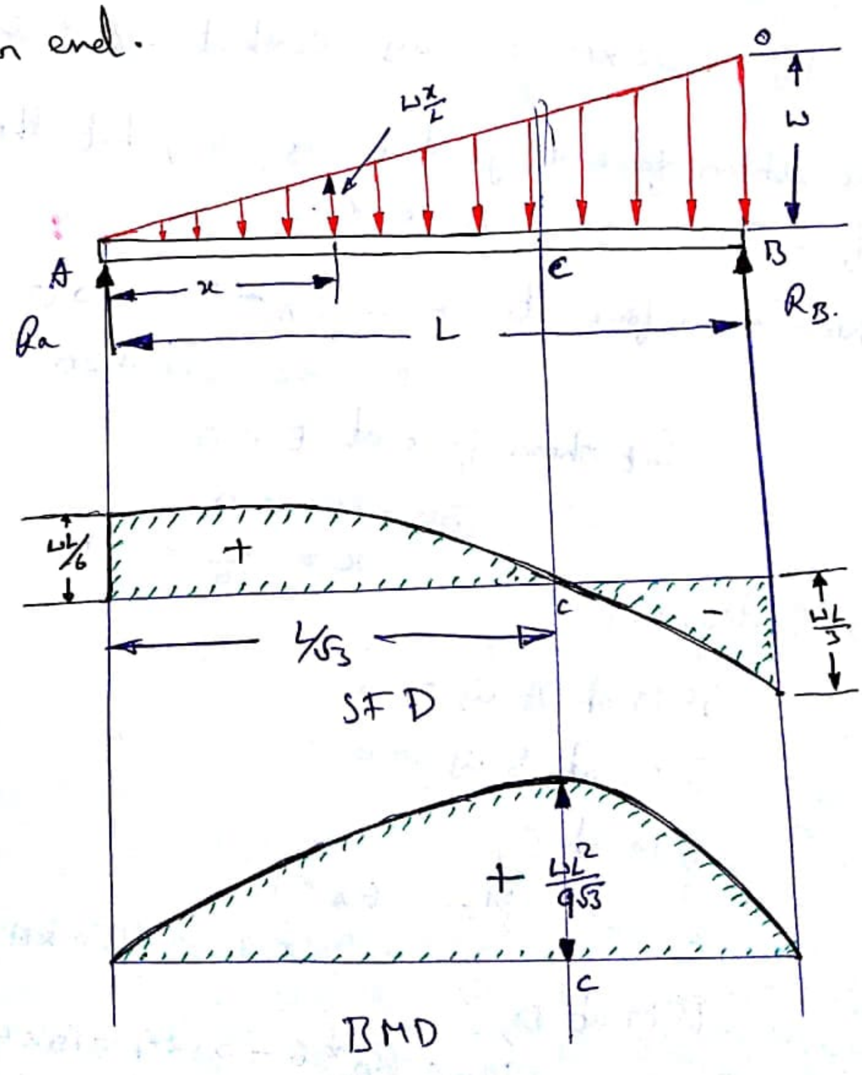
$$M_E = F_A \times 5 - 50(5-2) - 10 \times (5-2) \times \left(\frac{5-2}{2}\right)$$

$$= 80 \times 5 - 50 \times 3 - 10 \times 3 \times \frac{3}{2}$$

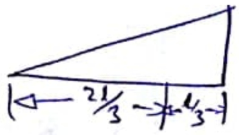
$$= 205 \text{ kNm}.$$

The B.M between C and D varies according to parabolic law reaching a maximum value of E

Shear force and B.M. Diagrams for a simply supported beam carrying a uniformly varying load from zero at one end to w per unit length at the other end.



Taking moments about A, we get



$$R_B \times L = \left(\frac{w.L}{2}\right)\frac{2}{3}L$$

$$\boxed{R_B = \frac{wL}{3}}$$

$$R_A + R_B = \frac{wL}{2}$$

$$R_A = \frac{wL}{2} - \frac{wL}{3} = \frac{3wL - 2wL}{6}.$$

$$\boxed{R_A = \frac{wL}{6}}$$

Considering any section x at a distance x from end A.
The shear force at x is given by.

$$F_x = R_A - \text{load on length } Ax$$

$$= \cancel{R_A}\ \frac{wL}{6} - \frac{wx}{L}\frac{x}{2}$$

$$\boxed{F_x = \frac{wL}{6} - \frac{wx^2}{2L}}$$

At A, x=0

$$F_A = \frac{wL}{6} - \frac{w}{2L} \times 0$$

$$\boxed{F_A = \frac{wL}{6}}$$

At B, x=L.

$$F_B = \frac{wL}{6} - \frac{w}{2\cancel{L}}(L)^2 = \frac{wL - 3wL}{6}$$

$$= -\frac{2wL}{6}$$

$$\boxed{F_B = -\frac{wL}{3}}$$

S.F is zero at somewhere b/w A & B

Let S.F be zero to a distance x from A

$$0 = \frac{wL}{6} - \frac{wx^2}{2L}$$

$$\frac{\cancel{w}x^2}{\cancel{2}L} = \frac{\cancel{w}L}{\cancel{6}3}$$

$$x^2 = \frac{L^2}{3}$$

$$\boxed{x = \frac{L}{\sqrt{3}}}$$

B.M diagram

The B.M is zero at A and B.

The BM at the section X at a distance x from the end A is given by

$$M_x = R_A x - \text{Load on length } Ax \cdot \frac{x}{3}$$

$$= \frac{wL}{6} \cdot x - \frac{wx^2}{2L} \frac{x}{3}.$$

$$= \frac{wL}{6} x - \frac{wx^3}{6L}$$

B.M varies between A and B according to cubic law.

The Max B.M at a distance $\frac{L}{\sqrt{3}}$ from end A.

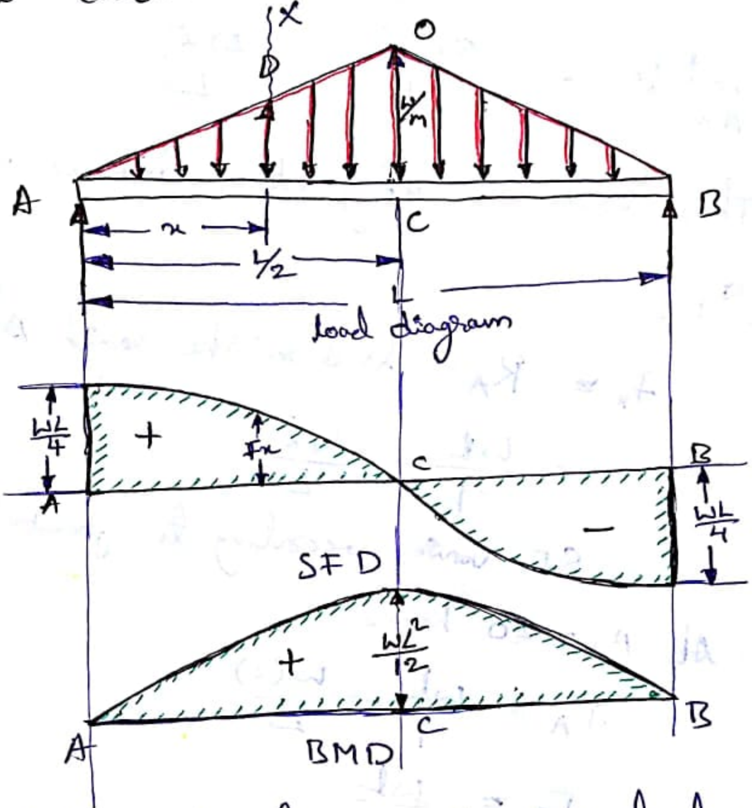
$$= \frac{wL}{6} \frac{L}{\sqrt{3}} - \frac{w}{6L} \frac{L^3}{3\sqrt{3}}$$

$$= \frac{wL^2}{6\sqrt{3}} - \frac{wL^2}{18\sqrt{3}} = \frac{wL^2}{\sqrt{3}} \left(\frac{1}{6} - \frac{1}{18}\right).$$

$$= \frac{wL^2}{\sqrt{3}} \left(\frac{3-1}{18}\right) = \frac{wL^2}{\sqrt{3}} \frac{\cancel{2}}{\cancel{18}\,9}.$$

$$\boxed{M_c = \frac{wL^2}{9\sqrt{3}}}$$

Shear force and bending moment diagrams for a simply supported beam carrying a uniformly varying load from zero at each end to 'w' per unit length at the centre.



Total load on the beam = Area of load diagram ABO

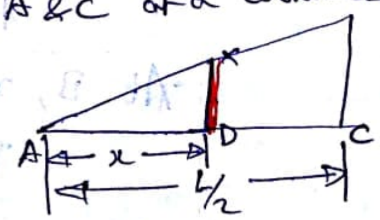
$$= \frac{AB \times CO}{2} = \frac{WL}{2}$$

$R_A = R_B =$ Half the total load.

$$R_A = \frac{WL}{4} \; ; \quad R_B = \frac{WL}{4}$$

Consider any section X b/w A & C at a distance x from end A

The rate of loading at X



$XD = \frac{W \times AD}{AC}$

$$W \times \frac{x}{\frac{L}{2}} = XD$$

$$= \frac{x}{\frac{L}{2}} \times W$$

$$= \frac{2W}{L} \cdot x$$

Now load on the length AX of the beam =
Area of load diagram AXD

$$= \frac{\frac{2wx}{L} \times x}{2}$$

Total load for AXD $= \dfrac{2wx^2}{2L} = \dfrac{wx^2}{L}$

This load is acting at a distance of $\frac{x}{3}$ from X

<u>SFD</u>

$F_x = R_A -$ load on the length Ax

$$= \frac{wL}{4} - \frac{wx^2}{L}$$

SF varies according to parabolic law

At A, $x = 0$ hence

$$F_A = \frac{wL}{4} - \frac{w(0)}{L}$$

$$F_A = \frac{wL}{4}$$

At C, $x = \frac{L}{2}$ hence

$$F_C = \frac{wL}{4} - \frac{w}{L}\left(\frac{L}{2}\right)^2$$

$$= \frac{wL}{4} - \frac{wL}{4}$$

$$F_C = 0$$

~~At B, $x = L$ hence~~

$$\cancel{F_B = \frac{wL}{4} - \frac{w}{L}L^2 = \frac{wL}{4} - wL}$$

$$\cancel{= -\frac{3wL}{4}}$$

Note:- As the beam loading varies from C to B so the shear force at B $= -R_B = -\dfrac{wL}{4}$

BM. diagram :

The bending moment is zero at A and B.

The B.M at x is given by;

$$M_x = R_A \cdot x - \text{load of length } Ax \cdot \frac{x}{3}$$

$$= \frac{wL}{4} \cdot x - \frac{wx^2}{L} \cdot \frac{x}{3}$$

$$= \frac{wL}{4} \cdot x - \frac{wx^3}{3L} \quad \text{———— (ii)}$$

At A, $x = 0$ hence.

$$M_A = 0$$

At C, $x = \frac{L}{2}$ hence

$$M_c = \frac{wL}{4} \frac{L}{2} - \frac{w}{3L} \left(\frac{L}{2}\right)^3$$

$$= \frac{wL^2}{8} - \frac{wL^2}{24}$$

$$= \frac{3wL^2 - wL^2}{24}$$

$$= \frac{\cancel{2}wL^2}{\cancel{24}\,12} = \frac{wL^2}{12}$$

The Maximum B.M. occurs at the centre of the beam, where SF becomes zero at centre

$$\text{Max B.M at C} \quad M_c = \frac{wL^2}{12}.$$

But total load on the beam, $W = \frac{wL}{2}$

$$\text{Max. B.M} = \frac{wL}{2} \cdot \frac{L}{6} = \frac{WL}{6}$$

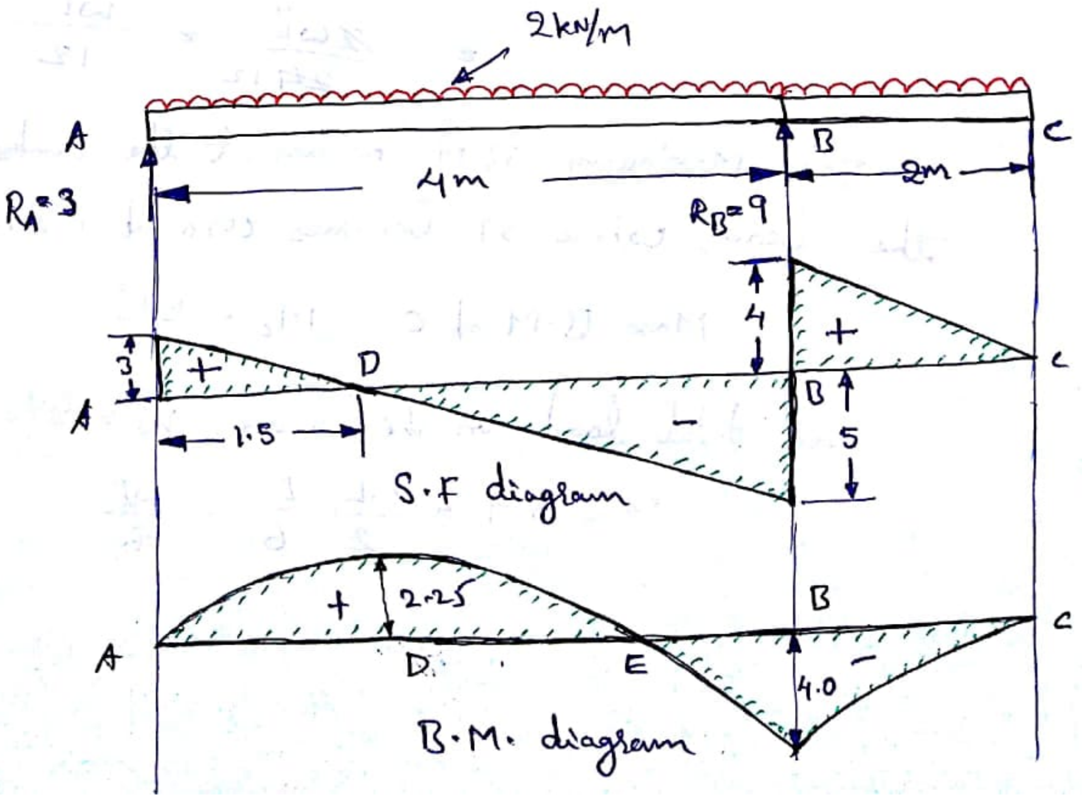
Shear force and Bending moment diagrams for overhanging beams

If the end portion of a beam is extended beyond the support, such beam is known as overhanging beams.

Point of contraflexure or Point of inflexion

It is the point where the B.M is zero after changing its sign from positive to negative or vice-versa.

Prob:- Draw the shear force and bending moment diagrams for the over-hanging beam carrying uniformly distributed load of 2 kN/m over the entire length as shown in figure. Also locate the point of contraflexure.



$$R_A + R_B = 2 \times 6$$

$$R_A + R_B = 12\,kN \quad \text{———— (i)}$$

Taking moments of all forces about A, we get

$$R_B \times 4 = 2 \times 6 \times \frac{6}{2}$$

$$= 36$$

$$\boxed{R_B = \frac{36}{4} = 9\,kN}$$ (substituting in eq i)

$$R_A = 12 - 9$$

$$\boxed{R_A = 3\,kN}$$

SFD

→ The shear force at any section between A and B at a distance x from A is given by

$$F_x = R_A - 2x \quad \text{———— (ii)}$$

$$\boxed{F_x = 3 - 2x}$$

At A, $x = 0$ hence

$$F_A = 0$$

At B, $x = 4$ hence

$$F_B = 3 - 2 \times 4 = -5\,kN$$

$$\boxed{F_B = -5\,kN}$$

SF varies according to straight line law b/w A & B where F_A is +ve and F_B is -ve, so SF is zero at some position b/w A & B,

Consider $F_x = 0$ in eq (ii)

$$0 = 3 - 2x$$

$$\boxed{x = 1.5\,m}$$

→ The SF at any section between B & C at a distance x from A is given by

$$F_x = R_A - 4 \times 2 + R_B - (x-4) \times 2$$

$$= 3 - 8 + 9 - 2(x-4)$$

$$\boxed{F_x = 4 - 2(x-4)}$$

At B, x = 4m hence.

$$F_B = 4 - 2(4-4)$$

$$\boxed{F_B = +4 \text{ kN}}$$

At C, x = 6m hence

$$F_C = 4 - 2(6-4)$$

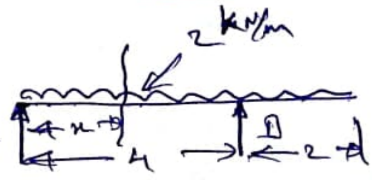
$$= 4 - 4$$

$$\boxed{F_C = 0}$$

Between B and C also SF varies by a straight line laws

BMD

→ The bending moment at any section between A and B at a distance x is given by,



$$M_x = R_A \times x - 2 \times x \times \frac{x}{2}$$

$$\boxed{M_x = 3x - x^2}$$

At A, x = 0

$$M_A = 3 \times 0 - 0^2$$

$$\boxed{M_A = 0}$$

At B, x = 0

$$M_B = 3 \times 4 - 4^2$$

$$= 12 - 16$$

$$\boxed{M_B = -4 \text{ kNm}}.$$

Maximum bending moment occurs at D where SF is zero

At D, $x = 1.5$ m,

$$M_D = 3\times1.5 - (1.5)^2$$

$$= 4.5 - 2.25$$

$$\boxed{M_D = 2.25 \text{ kNm}}$$

The B.M.D varies according to parabolic law between A & B

→ the B.M at any section between B and C at a distance x is given by

$$M_x = R_A \times x - 2\times x\times\frac{x}{2} + R_B(x-4)$$

$$= 3x - x^2 + 9x - 36$$

$$\boxed{M_x = 12x - x^2 - 36}$$

At B, $x = 4$ hence

$\frac{12}{36}$

$$M_B = 12\times4 - 4^2 - 36$$

$$= 48 - 16 - 36$$

$$\boxed{M_B = -4 \text{ kNm}}$$

At C, $x = 6$ hence.

$$M_C = 12\times6 - 6^2 - 36$$

$$= 72 - 36 - 36$$

$$\boxed{M_C = 0}$$

Point of Contraflexure

This point is between A and B where BM is zero after changing its sign. But B.M at any section at a distance x from A between A and B is given by

$$M_x = 3x - x^2$$

$M_x = 0$ at point of contraflexure

$$0 = 3x - x^2$$

$$x(3-x) = 0$$

$$3 - x = 0$$

$$\boxed{x = 3m}$$

Prob: A beam of length 12m is simply supported at two supports which are 8m apart, with an overhang of 2m on each side as shown in figure - the beam carries a concentrated load of 1000N at each end. Draw S-F and B.M diagrams.

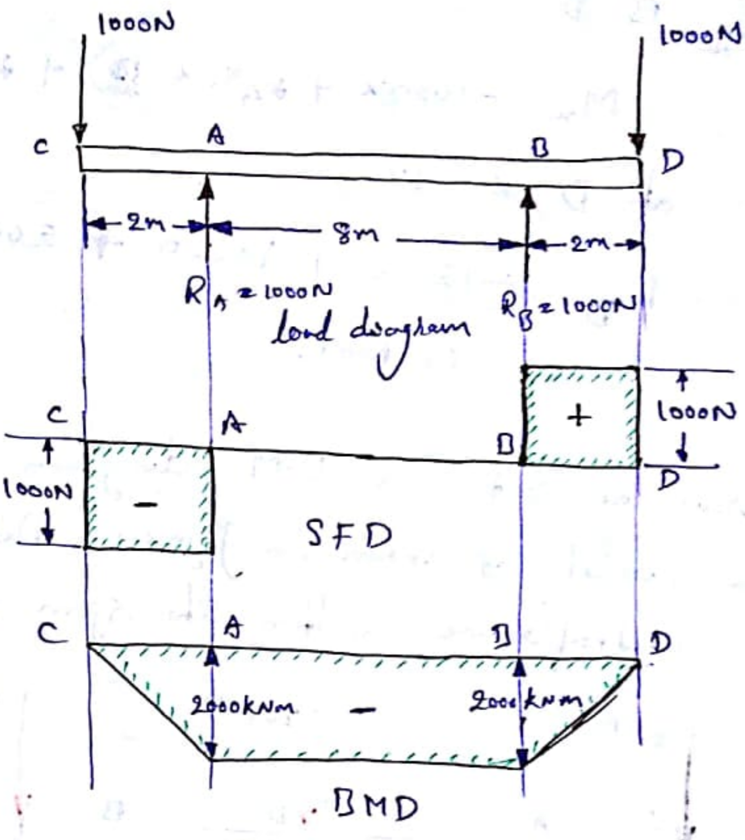
$$R_A + R_B = \frac{1000 + 1000}{2}$$

$$= 1000N$$

SFD

S-F at C $= -1000N$

S-F remains constant to the next load $= -1000N$ between C and A



S.F at A $= -1000 + R_A$

$= -1000 + 1000 = 0$

S.F remains constant between A and B

S.F at B $= 0 + 1000$

$= +1000N$

S.F remains constant between B and D (1000 N).

<u>B.M.D</u>

b/w C & A B.M at C $= M_x = -1000x$.

at C $x = 0$

$M_C = 0$

at A $x = 2$

$M_A = -2000$ kNm.

b/w A & B

$M_x = -1000x + R_A \times (x-2)$

at B, $x = 10$

$M_B = -10000 + 1000 \times 8$

$= -2000$ Nm.

b/w B & D

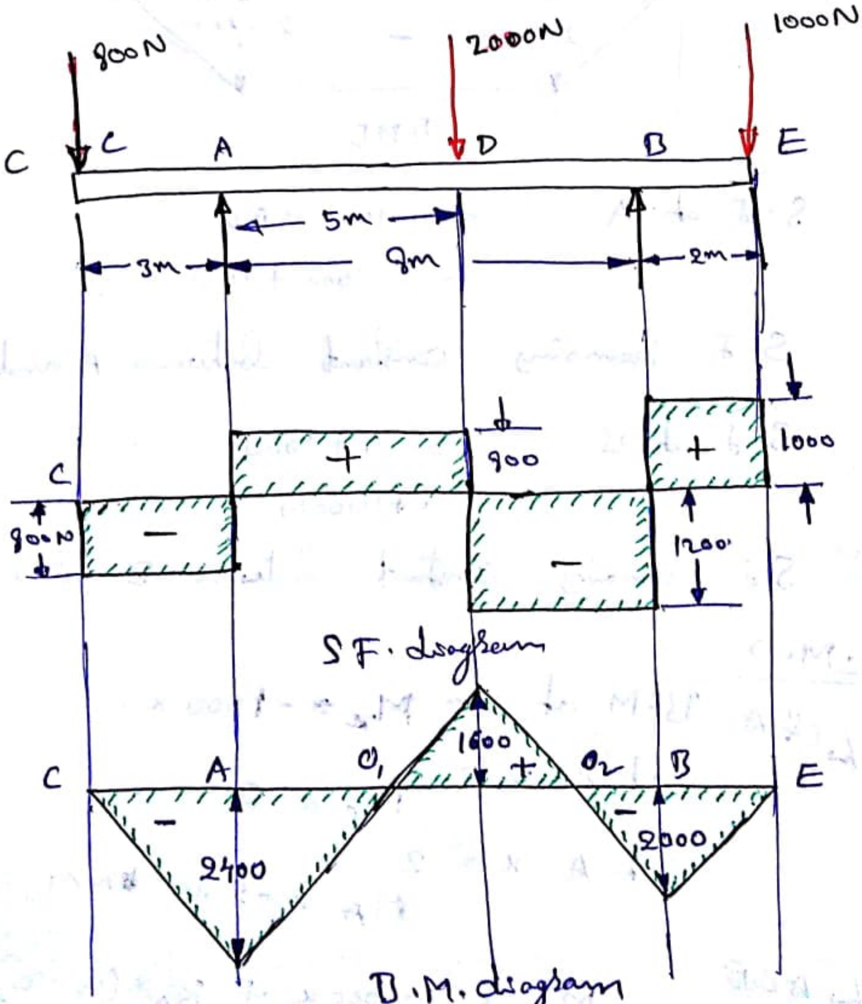
$$M_x = -1000x + R_A \times (x-2) + R_B \times (x-10)$$

at D, $x = 12$

$$M_D = -12000 + 10000 + 2000$$

$$= 0 \text{ NM}.$$

Prob:- Draw the S.F and B.M diagrams for the beam, which is loaded as shown in figure. Determine the points of contraflexure within the span AB.



$R_a + R_b = 800 + 2000 + 1000$

$R_a + R_b = 3800\ N$

Taking moments about A, we have

$R_B \times 8 + 800 \times 3 = 2000 \times 5 + 1000 \times 10$

$8R_B + 2400 = 10000 + 10000$

$8R_B = 17,600$

$\boxed{R_B = 17,600/8 = 2200\ N}$

$R_A = 3800 - 2200$

$\boxed{R_A = 1600\ N}$

S.F diagram

S.F at C $= -800\ N$

SF $-800\ N$ remaining constant between C & A

S.F at A $= -800 + 1600$

$\boxed{F_A = +800\ N}$

SF $+800\ N$ remaining constant between A & D.

S.F at D $= -800 + 1600 - 2000$

$\boxed{F_D = -1200\ N}$

SF $-1200\ N$ remaining constant between D & B

S.F at B $= -800 + 1600 - 2000 + 2200$

$\boxed{F_B = +1000\ N}$

SF $+1000\ N$ remaining constant between B & E

$\boxed{F_E = +1000\ N}$

BM diagram b/w C&A

BM at C =

$M_x = -800 \times x$

at C, $x = 0$

$\boxed{M_C = 0}$

at A, $x = 3m$

$M_A = 800 \times 3$

$\boxed{M_A = -2400 Nm.}$

BM diagram b/w A&D

$M_x = -800x + R_A(x-3)$

at D, $x = 8m$

$M_D = -800 \times 8 + 1600 \times 5.$

$\boxed{M_D = +1600 \text{ Nm}}$

BM diagram b/w D & B

$M_x = -800x + R_A(x-3) - 2000(x-8)$

at B, $x = 11m.$

$M_B = -800 \times 11 + 1600(11-3) - 2000(11-8)$

$= -8800 + 12800 - 6000$

$\boxed{M_B = -2000 \text{ Nm}}$

at E, $x = 13m.$

$M_A = -800 \times 13 + 1600(13-3) - 2000(13-8)$

$\boxed{M_A = 0}$

Point of Contraflexture

Then bending Moment at O_1 is zero (b/w A&D)

$$M_x = -800x + R_A(x-3)$$

$$0 = -800x + 1600(x-3)$$

$$= -800x + 1600x - 4800$$

$$= +800x - 4800$$

$$x = \frac{4800}{800}$$

$$\boxed{x = 6\,m.}$$

O_1 is at a distance of 6m from end C

BM at O_2 is zero (b/w D & B)

$$M_x = -800x + R_A(x-3) - 2000(x-8)$$

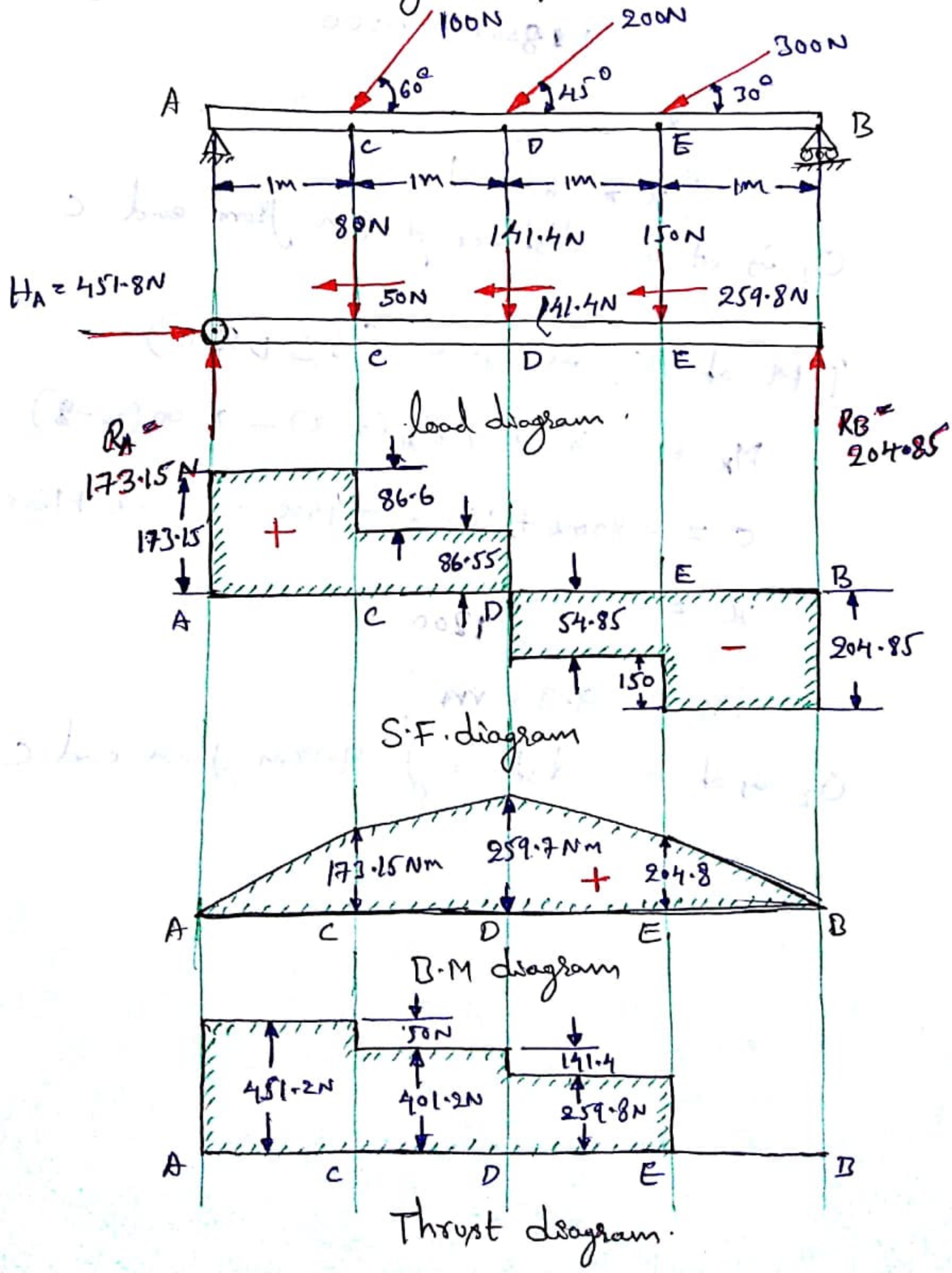
$$0 = -800x + 1600x - 4800 - 2000x + 16000$$

$$x = 11{,}200/1200$$

$$\boxed{x = 9.33\,m}$$

O_2 is at a distance of 9.33m from end C

Prob:- A horizontal beam AB of length 4m is hinged at A and supported on rollers at B. The beam carries inclined loads of 100N, 200N and 300N inclined at 60°, 45° and 30° to the horizontal as shown in figure. Draw the SF and B.M and thrust diagrams for the beam.



When an inclined load is acting on the beam the horizontal and vertical components should be calculated.

Note:- Due to vertical loads the SF and bending moment is generated.

Due to horizontal load, Thrust force is generated.

Loads at C: Horizontal component = $100 \cos 60 = 50N$
vertical component = $100 \sin 60 = 86.6N$

Loads at D: Horizontal component = $200 \cos 45 = 141.4N$
vertical component = $200 \cos 45 = 141.4N$

Loads at E: Horizontal component = $300 \cos 30 = 259.8N$
vertical component = $300 \sin 30 = 150N$

Beam is supported on rollers at B, hence roller support at B will not provide any horizontal reaction. the horizontal reaction will be only provided by hinged end A.

Let H_A = Horizontal reaction at A
= Sum of all horizontal components
= $50 + 141.4 + 259.8$

$H_{AC} = 451.20N$
$H_{CD} = 401.20N$
$H_{DE} = 259.8N$
$H_{EB} = 0N$

Thrust load.

SFD

To find the reactions R_A and R_B, take the moments of all forces about A,

$$R_B \times 4 = 86.6 \times 1 + 141.4 \times 2 + 150 \times 3$$

$$= 819.4/4$$

$$\boxed{R_B = 204.85\,N}$$

$$R_A + R_B = 86.6 + 141.4 + 150$$

$$R_A = 378 - 204.85$$

$$\boxed{R_A = 173.15\,N}$$

SFD

SF ~~at~~ from A to C is

$$\boxed{F_A = 173.15\,N}$$

SF from C to D is

$$F_C = 173.15 - 86.6$$

$$\boxed{F_C = 86.55\,N}$$

SF from D to E is

$$F_D = 86.55 - 141.40$$

$$\boxed{F_D = -54.85\,N}$$

(Here the SF changes the sign from +ve to -ve)

SF from E to B is

$$F_E = 173.15 - 86.6 - 141.40 - 150$$

$$\boxed{F_E = -204.85\,N}$$

SF $-204.85\,N$ is constant from E to B.

$$\boxed{F_B = -204.85\,N}$$

BMD

Consider a section b/w A & C

$M_x = R_A x$

at A, $x=0$

$\boxed{M_A = 0}$

at C, $x=1$

$M_C = 173.15 \times 1$

$\boxed{M_C = 173.15 \text{ Nm.}}$

Consider a section b/w C & D

$M_x = R_A x - 86.6(x-1)$.

at D, $x=2$

$M_D = 173.15 \times 2 - 86.6(2-1)$

$= 346.3 - 86.6$

$\boxed{M_D = 259.7 \text{ Nm.}}$

Consider a section b/w D & E

$M_x = R_A x - 86.6(x-1) - 141.4(x-2)$

at E, $x=3$

$M_E = 173.15 \times 3 - 86.6(3-1) - 141.4(3-2)$

$= 519.45 - 173.2 - 141.4$

$\boxed{M_E = 204.8 \text{ Nm}}$

Consider a section b/w E & B

$M_x = R_A x - 86.6(x-1) - 141.4(x-2) - 150(x-3)$

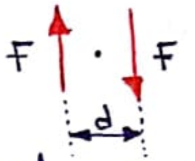
at B, $x=4$

$M_B = 692.6 - 259.8 - 282.8 - 150$

$\boxed{M_B = 0}$

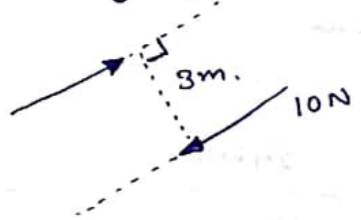
Couple and Applied Moments

A couple is two forces equal in magnitude opposite in sense parallel lines of action separated by a distance 'd'



- Moment due to couple Fd
- Same at any point on plane

ex:-



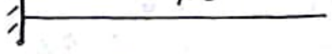
$$M = 10 \times 3 \text{ Nm}$$
$$= \underline{30 \text{ Nm}}$$

When a beam is subjected to a couple at a section, only the bending moment at the section of the couple changes suddenly in magnitude equal to that of the couple, But the SF doesn't change at the section of the couple as there is no change in load due to couple at the section.

To calculate the reactions, the magnitude of the couple is taken into account.

Sign Conventions

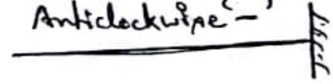
Anticlockwise '+'
clockwise '−'



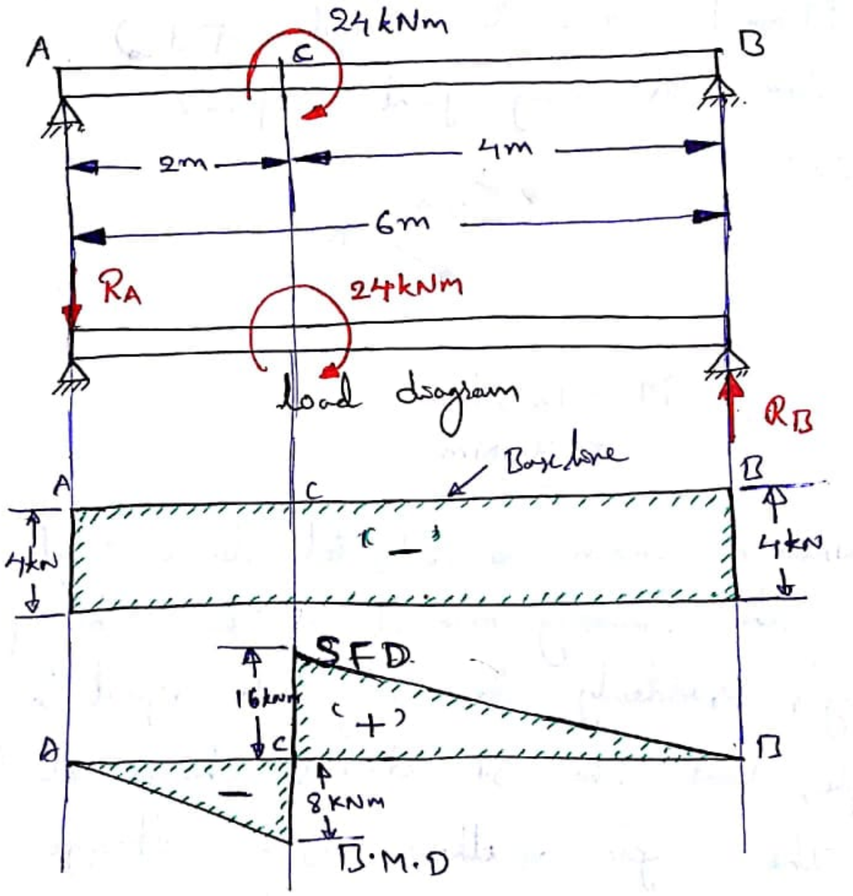
← If we consider forces or loads from right side

If we consider loads from left side →

clockwise '+'
Anticlockwise '−'



Prob:- A simply supported beam AB of length 6m. is hinged at A and B. It is subjected to a clockwise couple of 24kNm at a distance of 2m from the left end A. Draw the SF and B.M. diagrams.



To find reactions of R_A and R_B, take the moments about A.

$$R_B \times 6 - 24 = 0$$

$$R_B = 24/6$$

$\boxed{R_B = 4\,kN}$ ↑(upward due to '+' sign) take moments at B

$$R_A \times 6 + 24 = 0$$

$$R_A = -24/6$$

$$\boxed{R_A = -4} \downarrow \text{(downward due to -ve sign)}$$

SFD

S.F at A = $\boxed{F_A = -4\,kN}$

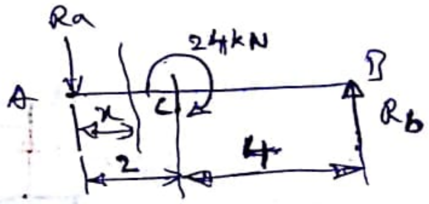
The SF remains constant between A and B.

$$\boxed{F_B = -4\,kN}$$

BMD

B.M at A & B = 0

Consider a section between A & C



$$M_x = R_a \times x.$$

at A, $x = 0$

$$\boxed{M_A = 0}$$

at C, $x = 2$

$$\boxed{M_c = -4 \times 2 = -8\,kNm}$$

As the moment 24kN is acting at 'C' so consider a section x between C and B, [There is a sudden change in BM at C due to couple]

$$M_x = R_a \times x + 24$$

at C, $x = 2$

$$M_c = -4 \times 2 + 24$$

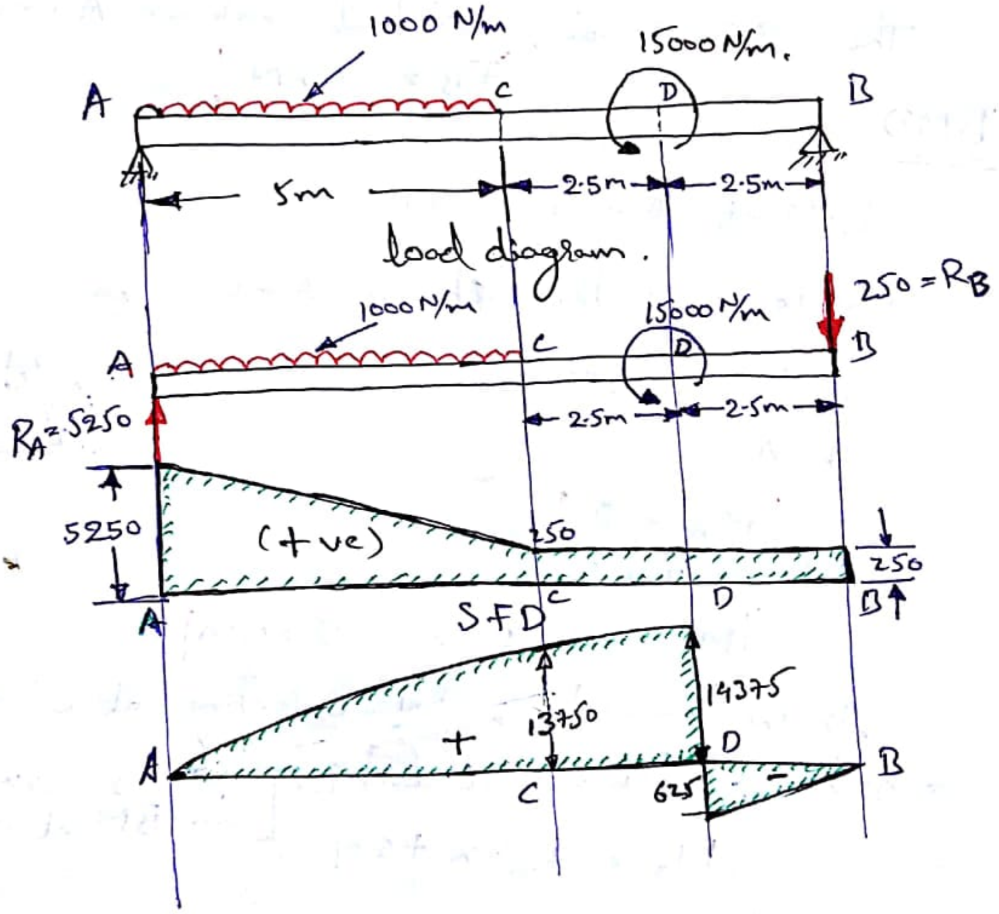
$$\boxed{M_c = -8 + 24 = -16\,kNm}$$

at B, $x = 6$

$$M_B = -4 \times 6 + 24$$

$$\boxed{M_B = 0}$$

Prob:- A beam 10 m long and simply supported at each end, has a uniformly distributed load of 1000 N/m extending from the left end upto the centre of the beam. There is also an anti-clockwise couple of 15 kNm at a distance of 2.5m from the right end. Draw the S.F and B.M diagrams.



To find reactions R_A and R_B, take the moments about 'A'

$$R_B \times 10 + 15000 - 1000 \times 5 \times \frac{5}{2} = 0$$

$$R_B \times 10 + 15000 - 12500 = 0$$

$$R_B \times 10 = -2500$$

$\boxed{R_B = -250}$ N ↓ (negative sign indicates the load acting downwards)

Taking moments about B

$$R_A \times 10 - 1000 \times 5 \times 7.5 - 15000 = 0$$

$$R_A = \frac{+52,50\cancel{0}}{1\cancel{0}}$$

$$\boxed{R_A = +5250\,N} \uparrow$$ (+ve sign indicate Reaction upwards)

SFD

Considering a section between A & C

$$F_x = -wx + R_A$$

at A, $x = 0$ $\quad = -1000 \times 0 + 5250$

$$F_A = +5250\,N$$

at C, $x = 5$ $\quad F_C = -1000 \times 5 + 5250\,N$

$$= 5000\,N + 5250\,N$$

$$= 250\,N$$

There is no load between C to B, So 250N is constant.

BMD

Consider a section b/w A & C

$$M_x = R_A x - 1000 \times \cancel{x} \times \frac{x}{2}$$

at A, $x = 0$

$$M_A = 0$$

at C, $x = 5$

$$M_C = 5250 \times 5 - 1000 \times \frac{25}{2}$$

$$= 26,250 - 12,500$$

$$= 13,750\ Nm$$

Consider a section between C&D

$$M_x = R_A x - 1000 \times 5 \times \left(x - \frac{5}{2}\right)$$

at C, $x = 5m$

$$= 5250 \times 5 - 1000 \times 5 \times \left(5 - \frac{5}{2}\right)$$

$$= 26{,}250 - 12500$$

$$M_C = 13{,}750 \text{ Nm}$$

at D, $x = 7.5m$

$$M_D = 5250 \times 7.5 - 1000 \times 5 \times \left(7.5 - \frac{5}{2}\right)$$

$$= 39{,}375 - 25000$$

$$M_D = 14{,}375 \text{ Nm}$$

Consider a section between D&B

$$M_x = R_A x - 1000 \times 5 \times \left(x - \frac{5}{2}\right) - 15000 = 0$$

at D, $x = 7.5m$.

$$M_D = 5250 \times 7.5 - 1000 \times 5 \times \left(7.5 - \frac{5}{2}\right) - 15000$$

$$= 39{,}375 - 25000 - 15000$$

$M_D = -625$ Nm Considering Moment at D.

at B, $x = 10m$

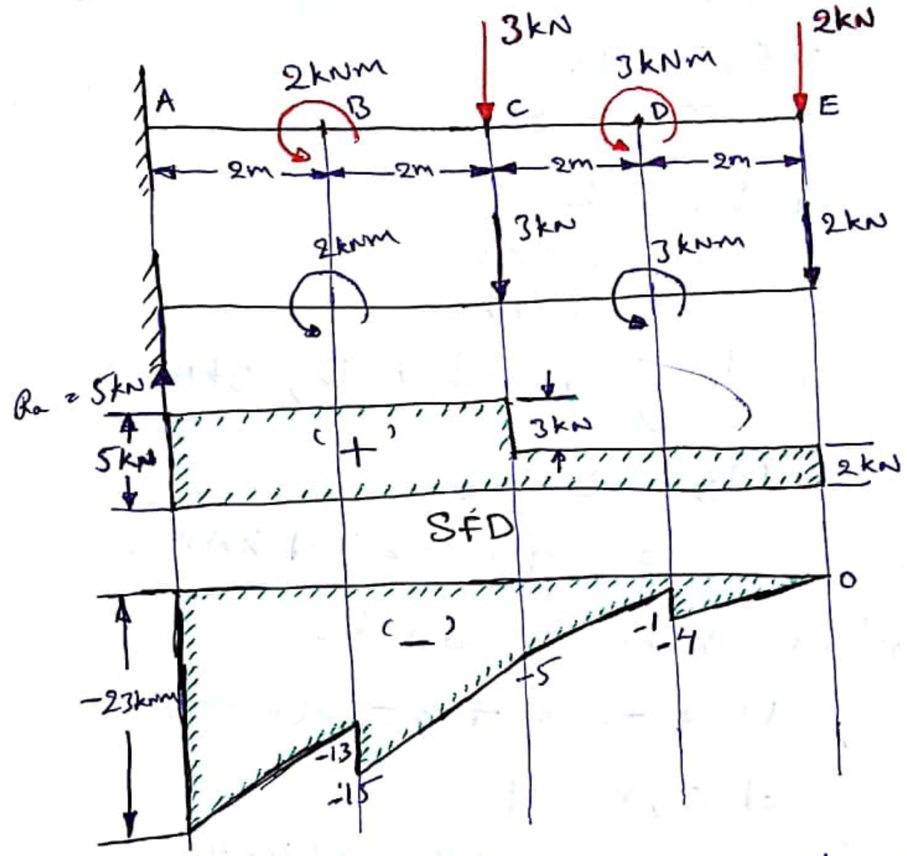
$$M_B = 5250 \times 10 - 1000 \times 5 \times \left(10 - \frac{5}{2}\right) - 15000$$

$$= 52500 - 37{,}500 - 15000$$

$$= 52500 - 52{,}500$$

$$M_B = 0$$

Prob: Find the reaction at the fixed end of the cantilever loaded as shown in figure. Draw the shear force and the bending moment diagrams.



To find reactions R_A

Total vertical loads on cantilever,

$= 3 + 2 = 5\,kN$

$R_a = 5\,kN$ (upwards)

SFD

$F_E = 2\,kN$, $F_D = 2\,kN$

F_E is constant from E to C.

$F_C = 2+3 = 5\,kN$

F_c is $5\,kN$ constant from C to A

F_A & $F_B = 5\,kN$

BMD

Consider a section b/w E & D.

$$M_x = -2 \times x$$

at E, $x = 0$

$$M_E = 0$$

at D, $x = 2$

$$M_D = -2 \times 2$$
$$= -4 \text{ kNm}$$

Considering moment at D, 3 kNm

$$M_D = -2 \times 2 + 3$$
$$= -4 + 3 = -1 \text{ kNm}.$$

Consider a section b/w C & B.

$$M_x = -2 \times x + 3 - 3(x-4)$$

at C, $x = 4$

$$M_C = -2 \times 4 + 3 - 3(4-4)$$
$$= -8 + 3 - 0$$
$$= -5 \text{ kNm}.$$

at B, $x = 6$

$$M_B = -2 \times 6 + 3 - 3(6-4)$$
$$= -12 + 3 - 6$$
$$= -15 \text{ kNm}.$$

Considering moment at B, 2 kNm.

$$M_B = -2 \times 6 + 3 - 3(6-4) + 2$$
$$= -15 + 2$$
$$= -13 \text{ kNm}.$$

Considering a section between A & B.

$M_x = -2\times x + 3 - 3(x-4) + 2$

at A, $x = 8\,m$

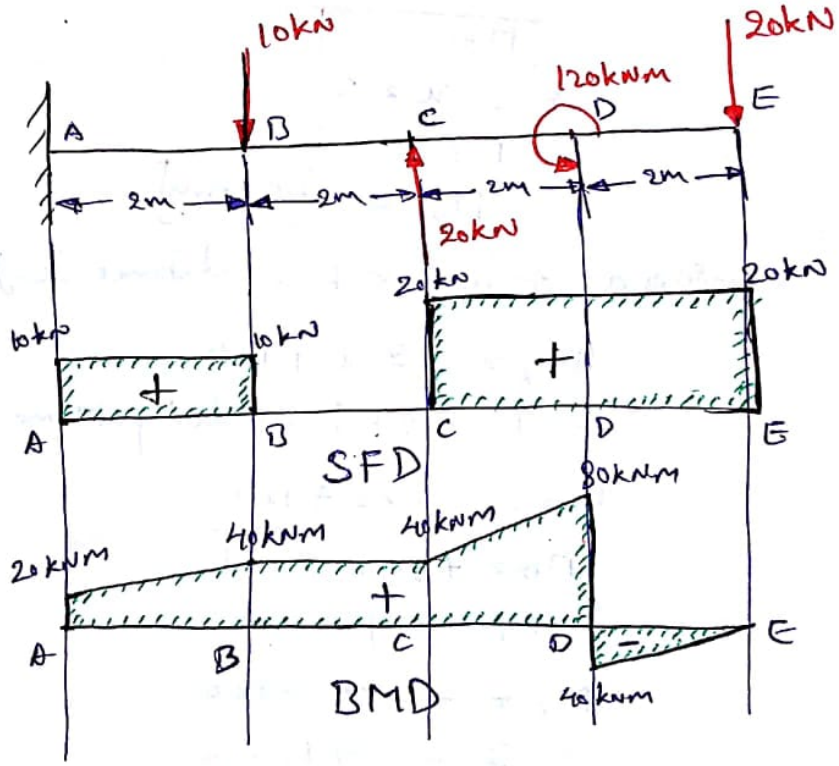
$M_A = -2\times 8 + 3 - 3(8-4) + 2$

$= -16 + 3 - 12 + 2$

$= -28 + 5$

$= -23\ kNm$

Prob:-



SFD

$F_E = +20\ kN$

S.F is constant b/w E to C

$F_C = 20 + 20$

$= +40\ kN$

S.F is constant b/w C to B

$F_B = 20 - 20 + 10\,kN$

SF $F_B = +10\,kN$

SF is constant from B to A.

<u>BMD</u>:-

Consider a section x at a distance x from E b/w D&E

$M_x = -20x$

at E, $x = 0$

$\boxed{M_E = 0}$

at D, $x = 2$

$M_D = -20 \times 2$

$\boxed{M_D = -40\,kNm}$

Consider a section x at a distance x from E b/w C&D

$M_x = -20x + 120$

at D, $x = 2$; considering Moment at D

$M_D = -20 \times 2 + 120$

$\boxed{M_D = +80\,kNm}$

at C, $x = 4$

$M_C = -20 \times 4 + 120$

$= -80 + 120$

$\boxed{M_C = +40\,kNm}$

consider a section x at a distance x from E b/w B&C

$M_x = -20x + 120 + 20(x-4)$

at C, $x = 4$

$\boxed{M_C = +40\,kNm}$

at B, $x = 6$

$M_B = -20 \times 6 + 120 + 20 \times 2$

$= -120 + 120 + 40$

$\boxed{M_B = +40\,kNm}$

Consider a section 'x' between A & B at a distance x from E

$$M_x = -20x + 120 + 20(x-4) - 10(x-6)$$

at A, $x = 8$

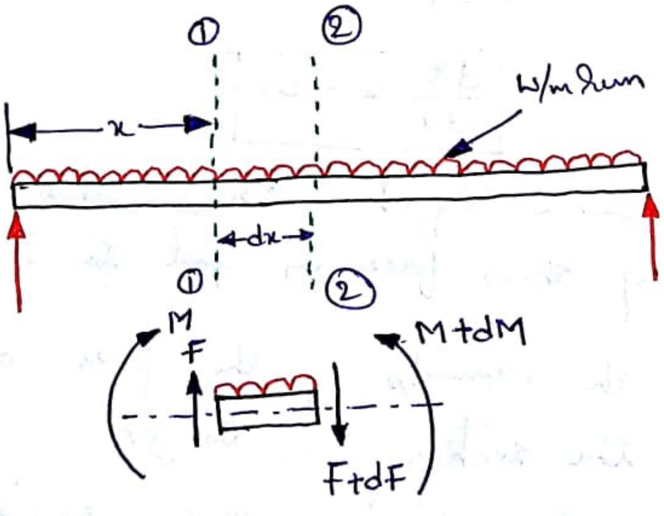
$$M_A = -160 + 120 + 80 - 20$$

$$\boxed{M_A = +20 \text{ kNM}}$$

Relations between Load, Shear Force and Bending Moment

Let a beam carrying a uniformly distributed load of w per unit length.

Consider the equilibrium of the portion of the beam between section 1-1 and 2-2. This portion is at a distance of x from left support and is of length dx



Let F = Shear force at the section 1-1,
F+dF = Shear force at the section 2-2,
M = Bending moment at the section 1-1,
M+dM = Bending moment at the section 2-2,

The forces and moments acting on the length 'dx' of the beam are:

i) The force F acting vertically up at the section 1-1

ii) The force F+dF acting vertically downwards at the section 2-2.

iii) The load w×dx acting downwards

iv) The moments M and (M+dM) acting at section 1-1 and section 2-2 respectively.

The portion of the beam of length 'dx' is in equilibrium. Hence resolving the force acting on this part vertically, we get.

$$F - w\cdot dx - (F + dF) = 0$$

$$-dF = w\cdot dx$$

$$\boxed{\frac{dF}{dx} = -w}$$

The above equation shows that the rate of change of shear force is equal to the rate of loading.

Taking the moments of the forces and couples about the section 2-2, we get.

$$M - w\cdot dx\cdot\frac{dx}{2} + F\cdot dx = M + dM$$

or $$-\frac{w\,(dx)^2}{2} + F\cdot dx = dM$$

Neglecting the higher powers of small quantities, we get.

$$F \cdot dx = dM$$

$$F = \frac{dM}{dx} \quad \& \quad \frac{dM}{dx} = F$$

The above equation shows that the rate of change of bending moments is equal to the shear force at the section.